North American Journal of Economics and Finance 69 (2024) 102015

Contents lists available at ScienceDirect

North American Journal of Economics and Finance

journal homepage: www.elsevier.com/locate/najef

Information sharing in a perfectly competitive market[☆]

Yaqing Yang[a,b], Youcheng Lou[a,*]

[a] Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, China
[b] University of Chinese Academy of Sciences, Beijing 100049, China

ARTICLE INFO

JEL classification:
D82
G14

Keywords:
Rational expectations equilibrium
Information sharing
Network connectedness
Welfare

ABSTRACT

We consider a large rational expectations economy where traders can share information with each other via an information network and investigate the impact of network connectedness on market equilibrium outcomes. We find that in the equilibrium with endogenous information, increasing network connectedness increases the total information in the market and trading volume, improves market efficiency, and enhances liquidity if and only if the market is sufficiently informationally efficient. Additionally, we provide a necessary and sufficient condition on the monotonicity of traders' welfare over network connectedness. We also show that the implications in the baseline model also hold for some extensions.

1. Introduction

The emergence of novel social media platforms and technological innovations has markedly facilitated the dissemination of information, rendering it more accessible, expedient, and efficacious. The exchange of information among participants in the market through social networks exerts a pivotal influence on the outcomes of market equilibrium. Communicated information among investors impacts their beliefs regarding asset fundamentals, which decisively shape investors' optimal asset demand. This in turn impacts equilibrium prices which emerge as an endogenous consequence of the equilibrium-seeking mechanism that aligns aggregate demand with supply. The aim of this paper is to analyze the impact of network connectedness of communication networks on the incentives for information acquisition and traders' welfare, and the equilibrium statistics including market efficiency, liquidity and trading volume.

We consider a single-period model and there is one risky asset and one riskless asset in the market. There is also noise trade in the market to prevent prices from being fully revealing. Each trader initially has one private signal related to the asset payoff with a signal-dependent cost, and can also share information with other traders in their network. Traders' preferences are represented by negative exponential utility functions, and traders make optimal demands based on their individual private information, the information received from their neighbors and the publicly observable price information. The equilibrium price of the risky asset is endogenously determined by market-clearing conditions.

We first show that there exists a unique symmetric linear endogenous equilibrium. Then, we show that traders' incentive for information acquisition is reduced as the network connectedness increases. The intuition is that when traders realize that they can receive more information from other traders, they have a temptation to free ride on the signals from other traders and reduce their information acquisition in the first place to save cost. Moreover, we show that the information of all traders or the total information

☆ This work was supported in part by the National Natural Science Foundation of China 71971208 and 72192804. The author order is random. Both authors are co-first authors of this paper.

* Corresponding author.
E-mail address: louyoucheng@amss.ac.cn (Y. Lou).

https://doi.org/10.1016/j.najef.2023.102015
Received 17 November 2022; Received in revised form 20 August 2023; Accepted 12 September 2023

in the market increases with network connectedness although each trader's incentive for information acquisition is lower. This is the first main contribution of the paper.

Additionally, we show that, in the endogenous equilibrium, increasing network connectedness improves market efficiency, increases trading volume, and enhances liquidity if and only if the market is sufficiently informationally efficient.

The second main contribution of the paper is that we provide necessary and sufficient conditions on the monotonicity of traders' welfare over network connectedness. Specifically, we show that a sign-keeping transformation of the derivative of the certainty equivalent is strictly decreasing in network connectedness. Finally, we consider three extensions of more general cost functions, an available public information in the market, and imperfect information sharing among traders, and show that the implications of network connectedness in the baseline model also hold for the three extensions.

Literature Review. Our work contributes to the recent literature on the implications of information networks on market equilibrium outcomes (Colla and Antonio (2010), Enward et al. (2019), Han and Yang (2013), Ozsoylev and Walden (2011), Walden (2019)). Our work is mostly related to the work of Han and Yang (2013). However, while our model shares the similar idea with these work that traders are socially connected with each other via an information network, our model diverges from their framework in a crucial aspect: while their model encompasses both informed and uninformed traders, our economy solely comprises informed traders.[1] This distinction gives rise to contrasting market implications regarding network connectedness. Han and Yang (2013) demonstrate that, in the presence of both informed and uninformed traders, network connectedness leads to a decrease in the total information available in the market. Consequently, increasing network connectedness adversely affects market efficiency, decreases trading volume, and enhances liquidity under the condition of sufficient informational inefficiency. Conversely, our findings paint a different picture. By focusing exclusively on informed traders, our model captures a distinct dynamic. The absence of uninformed traders naturally closes the free-riding channel associated with their signals. As a result, the overall crowding-out effect, as observed in Han and Yang (2013), weakens in our economy. This, in turn, leads to an intriguing outcome—the total information in our market actually increases with network connectedness. The disparity in overall market information immediately translates into divergent implications for market efficiency, trading volume, and liquidity since these market quality indexes depend on network connectedness and endogenous signal precision only in terms of their product, i.e., the total information in the market.[2]

In the case of endogenous information, Han and Yang (2013) initially consider a scenario where traders can either acquire an individual signal of fixed precision by incurring a constant cost or choose not to acquire any signal at all, without incurring any cost. Additionally, they explore an expanded setting in Subsection 5.2, where they introduce a signal-dependent cost function that closely resembles our own model's framework. While Han and Yang (2013) characterize the endogenous equilibrium by determining a fraction of informed traders where both informed and uninformed traders exhibit identical ex-ante welfare, as well as an optimal signal precision where traders have no incentive to deviate given the strategies of other traders, our economy solely revolves around determining the optimal signal precision. Despite the similarity in the modeling of information sharing, it is important to note that our economy is not a special case of Han and Yang (2013). Firstly, as the noises embedded in the shared signals decrease,[3] the endogenous fraction of informed traders in Han and Yang (2013) tends towards zero.[4] In contrast, in our economy, this fraction remains fixed at one, signifying a fundamental distinction in considering only informed traders. Secondly, in Han and Yang (2013), the extended model outlined in Subsection 5.2, the endogenous optimal signal precision is demonstrated to be independent of network connectedness. However, in our research, we find that the endogenous optimal signal precision is indeed influenced by network connectedness, illustrating a nuanced departure from their findings.

Our work is also closely related to the theoretical work of Colla and Antonio (2010), Ozsoylev and Walden (2011), Walden (2019) and the experimental work of Enward et al. (2019) that investigate the impact of information networks on market equilibrium outcomes.[5] Our model can be viewed as a special case of the economy of Ozsoylev and Walden (2011) with identical network connectedness across all traders, which considers a large economy in which traders share information via a sparse network structure with power law degree distributions. Colla and Antonio (2010) consider a dynamic finite-agent model where traders are locally informationally connected via a cyclical information network. Walden (2019) considers a dynamic large economy where information diffuses through a general information network. While these works study only the case of exogenous information, our focus is on the case of endogenous information. Enward et al. (2019) study the implications of social communication for traders' behavior as

[1] The exclusion of uninformed traders in our model is supported by two justifications. Firstly, the proliferation of social media has significantly enhanced the accessibility and speed at which agents can obtain information, thereby reducing search costs. Consequently, traders in contemporary developed markets are more likely to possess information, bolstering the rationale for our setting. Secondly, in financial markets, investors who lack essential information regarding asset payoffs face substantial challenges in sustaining their presence over extended periods. In fact, the economies without uninformed traders have been widely studied in the literature, for example, Colla and Antonio (2010), Han et al. (2016) and Walden (2019).

[2] Owing to the mathematical complexity of the expressions of trading volume and welfare in the economy of Han and Yang (2013), the authors analyze the effects of network connectedness on trading volume and welfare with the help of numerical examples. Here, we provide a theoretical analysis of a different model.

[3] Different from Han and Yang (2013) in which each (informed or uninformed) trader receives a noisy version of the signals of all informed traders, here traders are assumed to receive perfect signals from their neighbors without noises for simplicity. In Section 4.3, we will relax this assumption to more general cases of imperfect signal sharing and find that the main implications still hold.

[4] See Proposition 3 in Han and Yang (2013). The endogenous fraction of informed traders therein misses a term "$-\rho_y$" and the correct one should be $\mu^* = \frac{(2/(N\rho_y))((\rho_\epsilon-\rho_y)/(e^{2\gamma c}-1)-\rho_v)}{1+\sqrt{1+4(\rho_x/\gamma^2)((\rho_\epsilon-\rho_y)/(e^{2\gamma c}-1)-\rho_v)((\rho_\epsilon-\rho_y)/(N\rho_y)+(N-1)/N)^2}}$.

[5] There is also extensive empirical research on the effects of social interactions on investment decisions by investors (Brown et al. (2008), Feng and Seasholes (2004), Hong et al. (2004, 2005), Ivkovic and Weisbenner (2007), Ozsoylev and Walden (2014), Pool et al. (2015), Shiller and Pound (1989)); see Kuchler and Stroebel (2021) for a recent review.

well as market equilibrium statistics by designing an experiment. Different from the experimental method in Enward et al. (2019), our results are derived theoretically.

The paper is organized as follows. In Section 2 we introduce the model while in Section 3 we present the main results. Section 4 considers some extensions. Finally, Section 5 concludes. All proofs are in Appendix A. Some detailed proofs in the extensions can be found in the Online Supplemental Material.

2. The model

We consider a large noisy rational expectations economy where traders can share information with each other via an information network. It is a single-period model, and there are two assets in the market: one risky asset that pays $v \sim N(0, 1/\rho_v)$, $\rho_v > 0$, at the end of the trading period and one risk-free asset that pays zero interest with a perfectly elastic supply. The traders' utility from wealth w is

$$U(w) = -\exp\{-\gamma w\},$$

where $\gamma > 0$ is the absolute risk aversion coefficient of the traders. Consequently, the utility that trader i derives from the amount d_i invested in the risky asset is

$$U(W(d_i)) = -\exp\{-\gamma d_i(v-p)\},$$

where $W(d_i) = d_i(v-p)$ represents the terminal wealth of trader i, and p is the price of the risky asset that is publicly observable by all traders. To prevent the price from being fully revealing, noise traders also exist in the economy besides rational traders, where per-capita noise trade (supply) is denoted as $x \sim N(0, 1/\rho_x)$, $\rho_x > 0$, x is independent of other random variables in the economy.

The large economy consists of $G \in \mathbb{N}$ unconnected groups, denoted as $\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_G$, and each group contains N traders. The total number of traders in the large economy is correspondingly given by GN. We use the notation i, g to denote the ith trader in gth group $\mathcal{S}_g$. Each trader can receive a private signal by paying a cost that depends on signal precision. Let $s_{i,g} = v + \epsilon_{i,g}$ denote the private signal of trader i, g, where $\epsilon_{i,g} \sim N(0, 1/\rho_\epsilon)$, $\rho_\epsilon > 0$, and the noise $\{\epsilon_{i,g}\}_{i,g}$ are mutually independent and also independent of other random variables. The cost function $c(\cdot)$ of information acquisition is assumed to be twice continuously differentiable, strictly increasing, strictly convex, and satisfies the condition $c'(0) = 0$ (Han et al. (2016), Verrecchia (1982)).

A key feature of the large economy is that traders can share information with other traders in their group.[6] We assume that each trader i, g can observe $(k-1)$ signals of other traders perfectly in his/her group $\mathcal{S}_g$ via a k-cyclical graph besides his/her own signal $s_{i,g}$, where $1 \le k \le N$ is a positive integer. Let $\mathcal{K}_{i,g}$ denote the neighbor set of trader i, g ($i \in \mathcal{K}_{i,g}$), where jth trader in group $\mathcal{S}_g$ is one neighbor of trader i, g if and only if trader i, g can observe the signal of trader j, g. Consequently, the information set of trader i, g after information sharing with his/her neighbors is given by

$$\mathcal{F}_{i,g} = \{s_{j,g}, j \in \mathcal{K}_{i,g}, p\}.$$

According to the assumption of k-cyclical graphs, $|\mathcal{K}_{i,g}| = k$ for any $i = 1, \ldots, N$ and $g = 1, \ldots, G$. Similar to the parameter N of group size in Han and Yang (2013) and β of the average node degree in Ozsoylev and Walden (2011), we interpret k as "network connectedness". A higher network connectedness means that traders can share information with more traders.

Finally, we introduce the definition of noisy rational expectations equilibrium (NREE) with endogenous information, where we use $s_{i,g}(\rho)$ to highlight that the precision of signal $s_{i,g}$ is ρ, and $d_{i,g}(\rho)$ to denote the demand strategy of trader i, g, where ρ is the precision of his/her private signal.

Definition 1. An NREE with endogenous information is a collection $\left(\left(\rho^{i,g}_{\epsilon,k}, d^*_{i,g}(\rho^{i,g}_{\epsilon,k})\right)_{i=1,\ldots,N;g=1,\ldots,\infty}, p\right)$ of optimal signal precision, optimal demand strategies and a price such that

(i) $\left(d^*_{i,g}(\rho^{i,g}_{\epsilon,k})_{i=1,\ldots,N;g=1,\ldots,\infty}, p\right)$ constitutes an NREE with exogenous information $\left\{s_{i,g}(\rho^{i,g}_{\epsilon,k})\right\}_{i=1,\ldots,N;g=1,\ldots,\infty}$, i.e., for each $i = 1, \ldots, N$ and $g = 1, \ldots, \infty$, $d^*_{i,g}(\rho^{i,g}_{\epsilon,k})$ maximizes the conditional expected utility of trader i in group g,

$$d^*_{i,g}(\rho^{i,g}_{\epsilon,k}) \in \arg\max_d \mathbb{E}\left[U(W_{i,g}(d)) \middle| s_{j,g}(\rho^{j,g}_{\epsilon,k}), j \in \mathcal{K}_{i,g}, p\right],$$

and the market clears, i.e.,

$$\lim_{G\to\infty} \frac{1}{G} \sum_{g=1}^{G} \left[\frac{1}{N} \sum_{i=1}^{N} d^*_{i,g}(\rho^{i,g}_{\epsilon,k})\right] = x;$$

(ii) $\rho^{i,g}_{\epsilon,k}$ maximizes the ex-ante welfare of trader i in group g given the precision of other agents and resulting optimal strategies, i.e.,

$$\rho^{i,g}_{\epsilon,k} \in \arg\max_{\rho_i} \mathbb{E}\left[-\exp\left\{-\gamma\left[d^*_{i,g}(\rho_i)(v-p) - c(\rho_i)\right]\right\}\right]. \tag{1}$$

[6] In Section 4.2, we will consider a more general setting that there is an additional public signal available to all traders in the market and show that the main implications still hold.

In an NREE with endogenous information, each agent's signal precision is optimal given the signal precision of other agents and the resulting optimal strategies, and agents' strategies constitutes an NREE with exogenous information given the precision of all agents. Our economy shares the same market equilibrium as the economy of Ozsoylev and Walden (2011) with identical network connectedness across all traders. However, our focus is on the endogenous information acquisition, while Ozsoylev and Walden (2011) only consider the case of exogenous information.

3. Main results

As traders are equally risk averse and have the same cost function, we naturally consider a symmetric equilibrium: $\rho_{\epsilon,k}^{i,g} = \rho_{\epsilon,k}$ for all i, g. The following proposition shows that there exists a unique symmetric linear NREE in the economy with endogenous information.

Proposition 1. *There exists a unique symmetric linear NREE with endogenous information in which the equilibrium price is given by*[7]

$$p = \alpha_v v - \alpha_x x, \tag{2}$$

where

$$\alpha_v = \frac{\rho_\theta + k\rho_{\epsilon,k}}{\rho_v + \rho_\theta + k\rho_{\epsilon,k}},$$
$$\alpha_x = \frac{\gamma + k\rho_{\epsilon,k}\rho_x\gamma^{-1}}{\rho_v + \rho_\theta + k\rho_{\epsilon,k}},$$
$$\rho_\theta = \frac{(k\rho_{\epsilon,k})^2}{\gamma^2}\rho_x,$$

and $\rho_{\epsilon,k}$ is the unique positive root to the following equation

$$2c'(\rho)\left(\gamma^2\rho_v + \gamma^2 k\rho + \rho_x k^2\rho^2\right) = \gamma. \tag{3}$$

For notational convenience, we denote

$$z_k = k\rho_{\epsilon,k},$$

which represents the total signal precision of any trader (including his/her own information and that obtained from his/her neighbors) and can also be taken as a measure of total information in the market or in the price[8] because the network structure for information sharing is symmetric.

Proposition 2. *Increasing network connectedness reduces each trader's information acquisition, i.e., $\partial\rho_{\epsilon,k}/\partial k < 0$ and increases the total information in the market, i.e., $\partial z_k/\partial k > 0$.*

The first result in Proposition 2 can be intuitively understood as follows: When traders become aware that they can receive additional information from their peers, they may be tempted to rely on the signals provided by others and reduce their own information acquisition to save costs. This intuition aligns with the findings of Han and Yang (2013) and Enward et al. (2019). However, it remains unclear whether the total information in the market increases despite each trader acquiring signals of lower precision. The second result in Proposition 2 sheds light on this uncertainty. It reveals that the reduction in information acquisition is not significant enough to offset the increase in the total information available in the economy. This finding contradicts the results presented in Han and Yang (2013). The primary reason for this opposition lies in the fact that while both informed and uninformed traders can simultaneously free-ride on other traders' signals in Han and Yang (2013), the absence of uninformed traders in our model naturally eliminates the free-riding channel associated with their signals. As a result, the overall crowding-out effect weakens in our economy, leading to an increase in the total information available with greater network connectedness.

To illustrate this further, in Han and Yang (2013), informed traders recognize that increasing network connectedness brings about a greater number of both informed and uninformed traders who free-ride on their signals. However, uninformed traders do not share any information in return. Consequently, the incentive for informed traders to acquire information weakens. In contrast, our model lacks uninformed traders, and thus the private signals of informed traders are only subject to free-riding by other informed traders. This also weakens the incentive to acquire information. However, due to the absence of free-riding by uninformed traders, the weakening effect on the incentive to acquire information in our model is not as pronounced as in the presence of uninformed traders. Therefore, although increasing network connectedness still reduces each trader's information acquisition in our paper due to the crowding-out effect, the total information available in the market is enhanced, presenting a noteworthy departure from the findings of Han and Yang (2013).

Next, we analyze the impact of network connectedness on market equilibrium statistics and traders' welfare. Following Han and Yang (2013), we use $1/\operatorname{Var}[v|p]$ to measure the market efficiency and $1/\alpha_x$ to measure the liquidity. Market efficiency refers to

[7] There is no intercept in price function p since we assume that all random variables have mean zero for notational convenience.

[8] This can be seen from the relation $1/\operatorname{Var}[v|p] = \rho_v + z_k^2\rho_x/\gamma^2 \propto z_k^2$ as shown later.

how well prices reflect all available information in the market. A liquid market means that a noise trading shock is absorbed by the market without moving asset prices much. The trading volume is measured by $\mathbb{E}|d_{i,g}|$, and the (ex-ante) welfare is given by the term in (1). We have the following proposition on the implications.

Proposition 3. *For the unique symmetric linear NREE with endogenous information,*

(i) increasing network connectedness improves market efficiency and trading volume, and this enhances the liquidity if and only if the market is sufficiently informationally efficient in the sense that $\rho_\theta + 2\sqrt{\rho_\theta/\rho_x} > \rho_v/\gamma + 1/\rho_x$*;*

(ii) suppose that the cost function takes the form of $c(\rho) = \xi\rho^2, \xi > 0$,[9] *and denote*

$$A(k) := 4\xi\gamma^4 k\rho_{\epsilon,k}^3 + (2\gamma k\rho_x + 4\xi\gamma^6\rho_x^{-1} - \gamma\rho_x k^2 - 8\xi\gamma^4\rho_v)\rho_{\epsilon,k}^2 + \gamma^3(3-2k)\rho_{\epsilon,k} + \gamma^3\rho_v - \gamma^5\rho_x^{-1},$$

where $\rho_{\epsilon,k}$ *is the unique positive root to Eq. (3). Thus, increasing network connectedness initially increases and eventually decreases welfare if and only if* $A(1) > 0$ *and* $A(N) < 0$*, increases welfare if and only if* $A(N) \geq 0$*, and decreases welfare if and only if* $A(1) \leq 0$.[10]

The results in part (i) contrast with the finding of Han and Yang (2013) that in the endogenous NREE, increasing network connectedness harms market efficiency, decreases trading volume, and improves liquidity if and only if the market is sufficiently informationally inefficient. In addition, Han and Yang (2013) show that, for the extended model in Subsection 5.2, the optimal endogenous information precision is independent of network connectedness and the total information in the market (measured by ρ_θ in terms of our notation) decreases with network connectedness. The difference between the results in Han and Yang (2013) and ours is a consequence of (i) the decreasing total information in Han and Yang (2013) and the increasing total information in our model as shown by Proposition 2, and (ii) the equilibrium price and traders' demand strategies and, consequently, market efficiency, liquidity and trading volume depend on network connectedness k and the endogenous information signal $\rho_{\epsilon,k}$ only in terms of their product $k\rho_{\epsilon,k}$.

Part (ii) presents a necessary and sufficient condition on the monotonicity of traders' welfare over network connectedness. The absence of uninformed traders in the market allows us to have a tractable analysis for traders' welfare.[11] Part (ii) tells us that there are only three patterns of the monotonicity of welfare. In fact, each of the three patterns probably happens. For example, by some simple calculations we see that when ξ is very small, we have $A(1) > 0$ and $A(N) < 0$, and when ξ is very large, we have $A(N) \geq 0$ if $\rho_v > \gamma^2/\rho_x$ and $A(1) \leq 0$ if $\rho_v < \gamma^2/\rho_x$. To help understand the intuition behind the welfare result, let us see the expression (A.1) in the Appendix for welfare. We can see that there are two components $e^{2\gamma c(\rho_{\epsilon,k})}\operatorname{Var}\left[v|s_{j,g}, j \in \mathcal{K}_{i,g}, p_k\right]$ and $\operatorname{Var}(v - p_k)$ that determine the welfare. As network connectedness k increases, traders become better informed about the asset payoff, and thus the conditional variance $\operatorname{Var}\left[v|s_{j,g}, j \in \mathcal{K}_{i,g}, p_k\right]$ decreases and the endogenous price gets closer to the asset payoff, i.e., $\operatorname{Var}(v - p_k)$ decreases. Moreover, from Proposition 2, we know that $e^{2\gamma c(\rho_{\epsilon,k})}$ is decreasing in k. Hence, the two components $e^{2\gamma c(\rho_{\epsilon,k})}\operatorname{Var}\left[v|s_{j,g}, j \in \mathcal{K}_{i,g}, p_k\right]$ and $\operatorname{Var}(v - p_k)$ are both decreasing in k. The direction of traders' welfare with respect to network connectedness depends on which one of the two components is the dominant one.

4. Extensions

In this section, we consider three extensions to illustrate the robustness of our results in the baseline model.

4.1. General cost functions of information acquisition

In the baseline model, we analyze the welfare effects of network connectedness using quadratic cost functions. Here, we provide some numerical examples to illustrate the robustness of the welfare effects by considering more general cost functions $c(\rho) = \xi\rho^\ell$, $\ell \geq 2$.

The parameter settings are given as follows: The risk aversion coefficient $\gamma = 2$ (Han and Yang (2013)). We set $\rho_v = \rho_x = 5$, $\xi = 0.1$ in Panel (a) of Figs. 1–4; set $\rho_v = 40$, $\rho_x = 5$, $\xi = 10$ in Panel (b) of Figs. 1–4; and set $\rho_v = 3$, $\rho_x = 1$, $\xi = 10$ in Panel (c) of Figs. 1–4. In addition, the parameter ℓ in the cost function $c(\rho) = \xi\rho^\ell$ is, respectively, taken as 2, 3, 5, and 8 in the four figures.

The effects of network connectedness on traders' welfare are displayed in Figs. 1–4. Panel (a) of Figs. 1–4 show that traders' welfare initially increases and eventually decreases with network connectedness, Panel (b) of Figs. 1–4 show that traders' welfare strictly increases with network connectedness, and Panel (c) of Figs. 1–4 show that traders' welfare strictly decreases with network connectedness.

For the quadratic cost function $c(\rho) = \xi\rho^2$, we can numerically calculate that $A(1) > 0$ and $A(N) < 0$ under the given parameter values $\rho_v = \rho_x = 5$ and $\xi = 0.1$, that $A(N) > 0$ under the values $\rho_v = 40$, $\rho_x = 5$ and $\xi = 10$, and that $A(1) < 0$ under the values $\rho_v = 3$,

[9] In Section 4.1, we provide some numerical examples to illustrate that the welfare effects of network connectedness do not highly depend on the form of quadratic cost functions.

[10] When analyzing the welfare effects, the parameter of network connectedness k is treated as a continuous variable taking values in $[1, N]$ to facilitate the analysis.

[11] Due to the complexity of the expressions for welfare and trading volume in Han and Yang (2013), the authors analyze the implications with the help of numerical examples.

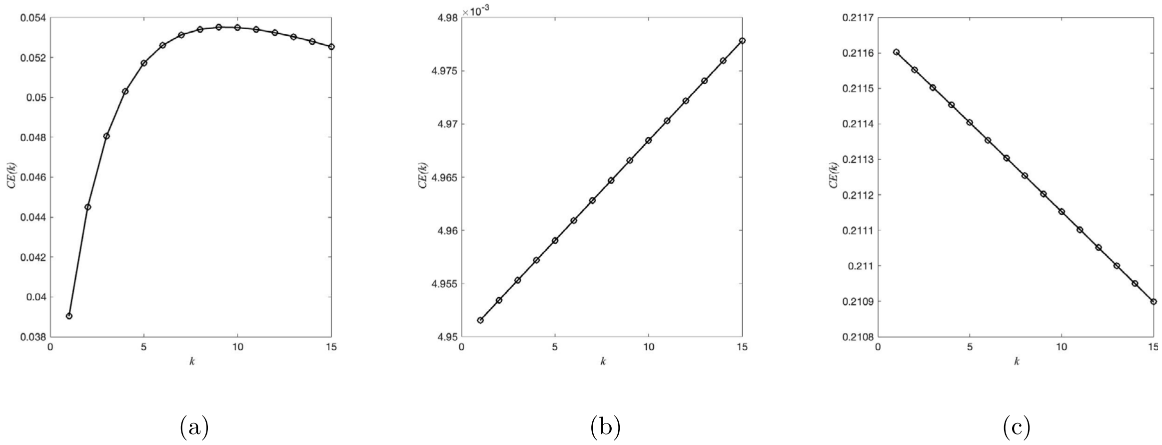


Fig. 1. Welfare effects of network connectedness for cost function $c(\rho) = \xi\rho^2$.

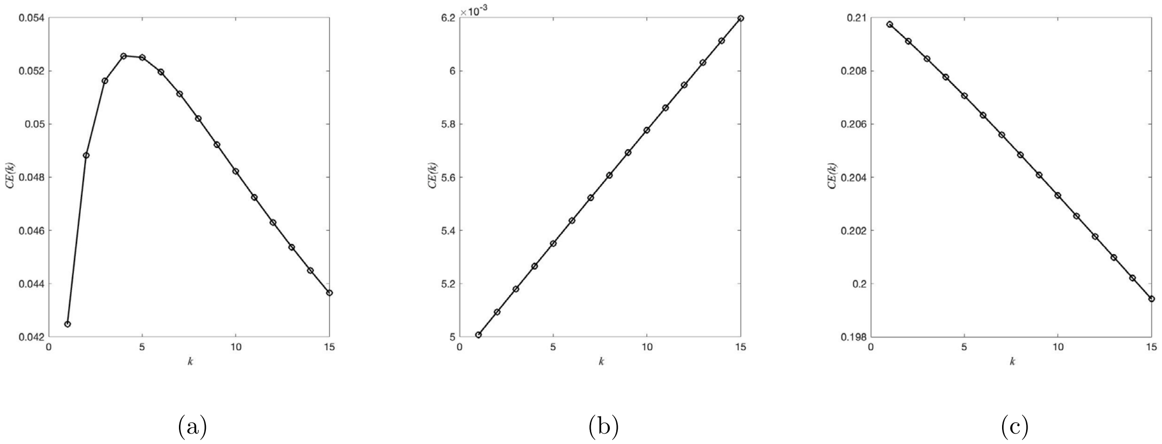


Fig. 2. Welfare effects of network connectedness for cost function $c(\rho) = \xi\rho^3$.

$\rho_x = 1$ and $\xi = 10$. Then, we can see that the numerical result in Fig. 1 is consistent with the theoretical result in Proposition 3 (ii). In addition, the numerical results in Figs. 2–4 for the cases of $\ell = 3$, $\ell = 5$, and $\ell = 8$ display a similar pattern to that in Fig. 1 for the case of $\ell = 2$; thus, the results in Proposition 3 (ii) do not highly depend on the form of the quadratic cost functions to some extent. The intuition is that the endogenous signal precision for a higher ℓ is lower, so that the endogenous cost for information acquisition and the welfare are not very sensitive to the parameter ℓ.

4.2. Public information

This subsection considers an extended setting where each trader can also observe a public signal related to the asset payoff besides his/her private signal. We denote the public information as

$$s = v + \zeta,$$

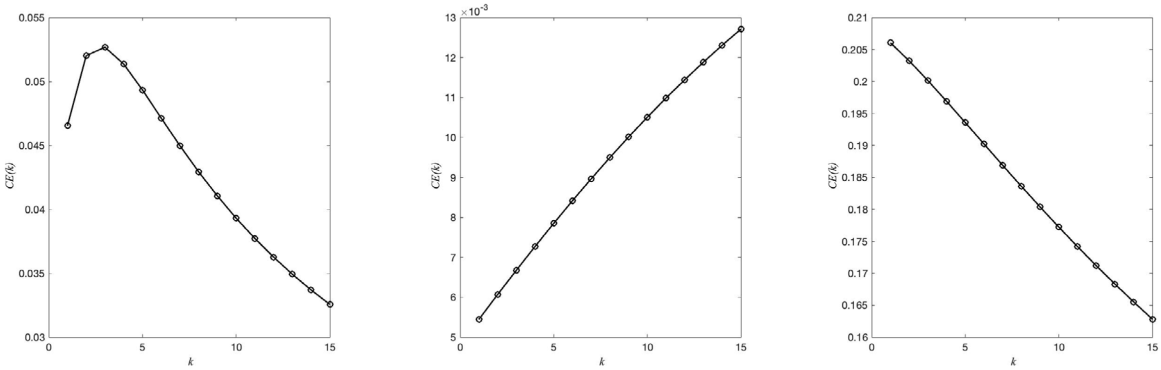
where $\zeta \sim N(0, 1/\rho_\zeta)$, $\rho_\zeta > 0$, is independent of other random variables in the model. The extended model reduces to the baseline model when $\rho_\zeta = 0$.

We still consider symmetric linear equilibrium and use a tilde to distinguish the variables here from that in the baseline model. We can show, in line with Proposition 1, that there exists a unique symmetric linear NREE with endogenous information in which the equilibrium price takes the same form as (2) with the replacement of $k\rho_{\epsilon,k}$ with $k\tilde{\rho}_{\epsilon,k} + \rho_\zeta$

$$\tilde{p} = \tilde{\alpha}_v v - \tilde{\alpha}_x x, \tag{4}$$

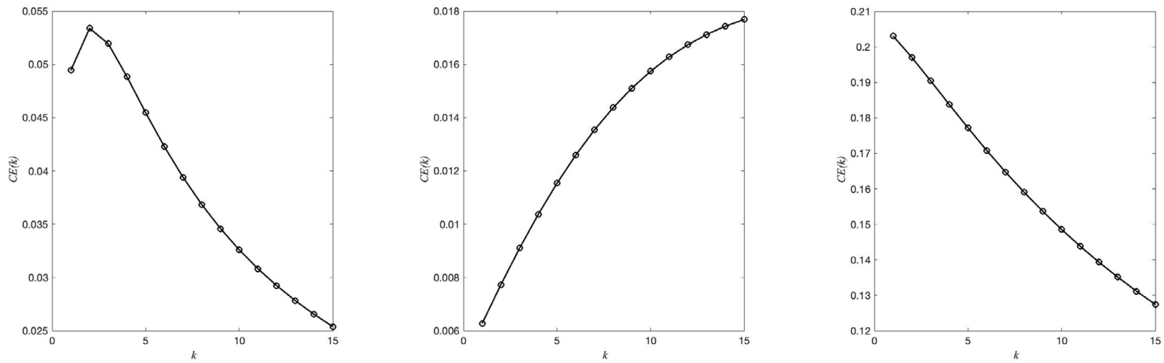
where

$$\tilde{\alpha}_v = \frac{\tilde{\rho}_\theta + k\tilde{\rho}_{\epsilon,k} + \rho_\zeta}{\rho_v + \tilde{\rho}_\theta + k\tilde{\rho}_{\epsilon,k} + \rho_\zeta}, \tag{5}$$



(a) (b) (c)

Fig. 3. Welfare effects of network connectedness for cost function $c(\rho) = \xi\rho^5$.



(a) (b) (c)

Fig. 4. Welfare effects of network connectedness for cost function $c(\rho) = \xi\rho^8$.

$$\tilde{\alpha}_x = \frac{\gamma + (k\tilde{\rho}_{\epsilon,k} + \rho_\zeta)\gamma^{-1}}{\rho_v + \tilde{\rho}_\theta + k\tilde{\rho}_{\epsilon,k} + \rho_\zeta}, \tag{6}$$

$$\tilde{\rho}_\theta = \frac{(k\tilde{\rho}_{\epsilon,k} + \rho_\zeta)^2}{\gamma^2}\rho_x, \tag{7}$$

and $\tilde{\rho}_{\epsilon,k}$ is the unique positive root to the following equation with variable $\tilde{\rho}$

$$2c'(\tilde{\rho})\left[\gamma^2\rho_v + \gamma^2(k\tilde{\rho} + \rho_\zeta) + \rho_x(k\tilde{\rho} + \rho_\zeta)^2\right] = \gamma. \tag{8}$$

Similar to z_k defined in the baseline model, let

$$\tilde{z}_k = k\tilde{\rho}_{\epsilon,k} + \rho_\zeta \tag{9}$$

denote the total information in the extended setting with a public signal. According to (8), we can see that increasing network connectedness reduces each trader's incentive for information acquisition, i.e, $\partial\tilde{\rho}_{\epsilon,k}/\partial k < 0$, and increasing the total information in the market, i.e., $\partial\tilde{z}_k/\partial k > 0$, which are consistent with the results in Proposition 2.

Also, in line with Proposition 3 (i), we can show that increasing network connectedness enhances market efficiency and trading volume, and increases liquidity if and only if the market is sufficiently informationally efficient in sense that $\tilde{\rho}_\theta + 2\sqrt{\tilde{\rho}_\theta/\rho_x} > \rho_v/\gamma + 1/\rho_x$.

We then consider the influence of network connectedness on traders' welfare, or equivalently, the certainty equivalent. The certainty equivalent in the presence of public information is given by

$$\widetilde{CE}(k) = \frac{1}{2\gamma}\log\left[\frac{\rho_x\tilde{z}_k^2 + 2\gamma^2\tilde{z}_k + \gamma^2\rho_v + \gamma^4\rho_x^{-1}}{\rho_x\tilde{z}_k^2 + \gamma^2\tilde{z}_k + \gamma^2\rho_v}\right] - c(\tilde{\rho}_{\epsilon,k}),$$

and under the assumption $c(\rho) = \xi\rho^2$ in the baseline model, we have

$$\frac{\partial \widetilde{CE}(k)}{\partial k} \propto 4\xi\gamma^6\rho_x^{-1}\tilde{\rho}_{\epsilon,k}^2 - 8\xi\gamma^4\rho_v\tilde{\rho}_{\epsilon,k}^2 + 4\xi\gamma^4\tilde{z}_k\tilde{\rho}_{\epsilon,k}^2 + 2\gamma\rho_x\tilde{z}_k\tilde{\rho}_{\epsilon,k} + 3\gamma^3\tilde{\rho}_{\epsilon,k} - 2\gamma^3\tilde{z}_k - \gamma\rho_x\tilde{z}_k^2 + \gamma^3\rho_v - \gamma^5\rho_x^{-1}$$
$$=: \tilde{A}(k).$$

Furthermore, we can show that $\tilde{A}(k)$ is strictly decreasing in k, i.e., $\partial\tilde{A}(k)/\partial k < 0$ for any $k \in [1, N]$. Please refer to the Online Supplemental Material for more details on the proofs of the above conclusions. Thus, similar to Proposition 3 (ii), we also have the following necessary and sufficient condition in the presence of public information: Increasing network connectedness initially increases and eventually decreases welfare if and only if $\tilde{A}(1) > 0$ and $\tilde{A}(N) < 0$, increases welfare if and only if $\tilde{A}(N) \geq 0$, and decreases welfare if and only if $\tilde{A}(1) \leq 0$.

4.3. Imperfect signal sharing

Instead of perfect information sharing, similar to Han and Yang (2013) here we assume that each trader can only receive a noisy version of his/her neighbors' signals. The signal of trader i in group g observed by his/her neighbors is denoted by

$$y_{i,g} = s_{i,g} + \eta_{i,g},$$

where $\eta_{i,g} \sim N(0, 1/\rho_\eta)$, $\rho_\eta > 0$, is the noise contained in the shared signals and η is independent of all other random variables in the economy. We have $y_{i,g} \sim N(0, 1/\rho_{y,k})$, where $\rho_{y,k} = \left(1/\hat{\rho}_{\epsilon,k} + 1/\rho_\eta\right)^{-1} > 0$. The generalized model is the same as the baseline model except that the information sharing among traders is imperfect. We still consider symmetric linear equilibrium, and to distinguish from the notations in the baseline model, here we use a hat to denote the variables.

Similar to Proposition 1, we can show that there exists a unique symmetric linear NREE with endogenous information in which the equilibrium price takes the same form as (2) with the replacement of $k\rho_{\epsilon,k}$ with $\hat{\rho}_{\epsilon,k} + (k-1)\rho_{y,k}$,

$$\hat{p} = \hat{\alpha}_v v - \hat{\alpha}_x x, \tag{10}$$

where

$$\hat{\alpha}_v = \frac{\hat{\rho}_\theta + \hat{\rho}_{\epsilon,k} + (k-1)\rho_{y,k}}{\rho_v + \hat{\rho}_\theta + \hat{\rho}_{\epsilon,k} + (k-1)\rho_{y,k}}, \tag{11}$$

$$\hat{\alpha}_x = \frac{\gamma + (\hat{\rho}_{\epsilon,k} + (k-1)\rho_{y,k})\rho_x\gamma^{-1}}{\rho_v + \hat{\rho}_\theta + \hat{\rho}_{\epsilon,k} + (k-1)\rho_{y,k}}, \tag{12}$$

$$\hat{\rho}_\theta = \frac{(\hat{\rho}_{\epsilon,k} + (k-1)\rho_{y,k})^2}{\gamma^2}\rho_x, \tag{13}$$

and $\hat{\rho}_{\epsilon,k}$ is the unique positive root to the following equation with variable $\hat{\rho}$

$$2c'(\hat{\rho})\left[\gamma^2\rho_v + \gamma^2(\hat{\rho} + (k-1)(1/\hat{\rho} + 1/\rho_\eta)^{-1}) + \rho_x(\hat{\rho} + (k-1)(1/\hat{\rho} + 1/\rho_\eta)^{-1})^2\right] = \gamma. \tag{14}$$

Let

$$\hat{z}_k = \hat{\rho}_{\epsilon,k} + (k-1)\rho_{y,k} = \hat{\rho}_{\epsilon,k} + \frac{k-1}{(\hat{\rho}_{\epsilon,k})^{-1} + \rho_\eta^{-1}} \tag{15}$$

denote the total information in the market. Thus, it follows from (14) that increasing network connectedness reduces each trader's incentive for information acquisition, i.e., $\partial\hat{\rho}_{\epsilon,k}/\partial k < 0$, and increases the total information in the market, i.e., $\partial\hat{z}_k/\partial k > 0$, which are in line with the results in Proposition 2 in the baseline model.

Similar to Proposition 3 (i) in the baseline model, we can also show that increasing network connectedness improves market efficiency and trading volume, and enhances the liquidity if and only if the market is sufficiently informationally efficient in sense that $\hat{\rho}_\theta + 2\sqrt{\hat{\rho}_\theta/\rho_x} > \rho_v/\gamma + 1/\rho_x$.

We then analyze the welfare (equivalently, the certainty equivalent) effects of network connectedness. We have the following form of certainty equivalent

$$\widehat{CE}(k) = \frac{1}{2\gamma}\log\left[\frac{\rho_x\hat{z}_k^2 + 2\gamma^2\hat{z}_k + \gamma^2\rho_v + \gamma^4\rho_x^{-1}}{\rho_x\hat{z}_k^2 + \gamma^2\hat{z}_k + \gamma^2\rho_v}\right] - c(\hat{\rho}_{\epsilon,k}).$$

Under the assumption $c(\rho) = \xi\rho^2$ in the baseline model, we further have

$$\frac{\partial \widehat{CE}(k)}{\partial k} \propto 12\xi\gamma^4\hat{z}_k\hat{\rho}_{\epsilon,k}^2 + 8\xi\gamma^2\rho_x\hat{z}_k^2\hat{\rho}_{\epsilon,k}^2 + 4\xi\gamma^6\rho_x^{-1}\hat{\rho}_{\epsilon,k}^2 + 2\gamma\rho_x\hat{z}_k\hat{\rho}_{\epsilon,k} + \gamma^3\hat{\rho}_{\epsilon,k} - \gamma\rho_x\hat{z}_k^2 - 2\gamma^3\hat{z}_k - \gamma^5\rho_x^{-1} + \gamma^3\rho_v$$
$$=: \hat{A}(k).$$

We can show that $\hat{A}(k)$ is strictly decreasing in k, i.e., $\partial\hat{A}(k)/\partial k < 0$ for any $k \in [1, N]$. Please refer to the Online Supplemental Material for more details on the proofs of the above conclusions. Thus, similar to Proposition 3 (ii), we have the following necessary and sufficient condition in the case of imperfect information sharing: Increasing network connectedness initially increases and eventually decreases welfare if and only if $\hat{A}(1) > 0$ and $\hat{A}(N) < 0$, increases welfare if and only if $\hat{A}(N) \geq 0$, and decreases welfare if and only if $\hat{A}(1) \leq 0$.

5. Conclusions

We analyze the impact of network connectedness on market equilibrium outcomes in term of market efficiency, liquidity, trading volume and welfare for a large rational expectations perfectly competitive market in which traders share information with each other via a cyclical graph. We find that the implications of information network on market quality with endogenous information are opposite to that in Han and Yang (2013). More specifically, we show that increasing network connectedness increases the total information in the market and trading volume, improves market efficiency, and increases liquidity if and only if the market is sufficiently informationally efficient. We also provide a necessary and sufficient condition on the monotonicity of welfare over network connectedness. Additionally, we consider three extensions of general information acquisition cost functions, an available public information and imperfect information sharing among traders to illustrate the robustness of the implications in the baseline model. An interesting future work is to investigate the effects of network connectedness on market equilibrium outcomes for imperfectly competitive markets, for example, the economy in Kyle (1989).

Declaration of competing interest

I have read the North American Journal of Economics and Finance's disclosure policy and have no conflicts of interest to disclose.

Appendix A

In this appendix, we provide the proofs for all the propositions.

Proof of Proposition 1

First, for the exogenous information $\{s_{i,g}(\rho_{\epsilon,k})\}_{i=1,\dots,N;g=1,\dots,\infty}$, following the standard procedures in Hellwig (1980), we can show that there exists a unique symmetric linear NREE, and the equilibrium price p has the form stated in the proposition.

We now show condition (ii) in Definition 1. We have

$$\mathbb{E}\left[-\exp\left\{-\gamma\left[d^*_{i,g}(\rho)(v-p)-c(\rho)\right]\right\}\right] = -\sqrt{\frac{e^{2\gamma c(\rho)}\operatorname{Var}\left[v\middle|s_{j,g}, j\in\mathcal{K}_{i,g},p\right]}{\operatorname{Var}(v-p)}} \tag{A.1}$$

$$= -\sqrt{\frac{1}{\operatorname{Var}(v-p)}\frac{e^{2\gamma c(\rho)}}{\rho_v+\rho_\theta+(k-1)\rho_{\epsilon,k}+\rho}}, \tag{A.2}$$

where (A.1) follows from (A.7) in Lou and Wang (2021), and (A.2) from (2) and the projection theorem for normal random variables. From (1) and (A.2), it suffices to show that there exists $\rho_{\epsilon,k}>0$ such that $\frac{e^{2\gamma c(\rho)}}{\rho_v+\rho_\theta+(k-1)\rho_{\epsilon,k}+\rho}$ achieves its minimum value at $\rho=\rho_{\epsilon,k}$.

We have

$$\frac{\partial\left[\frac{e^{2\gamma c(\rho)}}{\rho_v+\rho_\theta+(k-1)\rho_{\epsilon,k}+\rho}\right]}{\partial\rho} = \frac{e^{2\gamma c(\rho)}H(\rho)}{(\rho_v+\rho_\theta+(k-1)\rho_{\epsilon,k}+\rho)^2},$$

where $H(\rho)=2\gamma c'(\rho)(\rho_v+\rho_\theta+(k-1)\rho_{\epsilon,k}+\rho)-1$, and then

$$\begin{aligned}
\frac{\partial^2\left[\frac{e^{2\gamma c(\rho)}}{\rho_v+\rho_\theta+(k-1)\rho_{\epsilon,k}+\rho}\right]}{\partial\rho^2} &= \frac{\partial\left[\frac{e^{2\gamma c(\rho)}H(\rho)}{(\rho_v+\rho_\theta+(k-1)\rho_{\epsilon,k}+\rho)^2}\right]}{\partial\rho}\\
&= \frac{\left[2\gamma c''(\rho)e^{2\gamma c(\rho)}+4\gamma^2(c'(\rho))^2e^{2\gamma c(\rho)}\right]}{\rho_v+\rho_\theta+(k-1)\rho_{\epsilon,k}+\rho} - \frac{2\gamma c'(\rho)e^{2\gamma c(\rho)}}{(\rho_v+\rho_\theta+(k-1)\rho_{\epsilon,k}+\rho)^2}\\
&\quad - \frac{2\gamma c'(\rho)e^{2\gamma c(\rho)}}{(\rho_v+\rho_\theta+(k-1)\rho_{\epsilon,k}+\rho)^2} + \frac{2e^{2\gamma c(\rho)}}{(\rho_v+\rho_\theta+(k-1)\rho_{\epsilon,k}+\rho)^3}\\
&\propto (\rho_v+\rho_\theta+(k-1)\rho_{\epsilon,k}+\rho)^2\left[2\gamma c''(\rho)+4\gamma^2c'^2(\rho)\right]-4\gamma c'(\rho)(\rho_v+\rho_\theta+(k-1)\rho_{\epsilon,k}+\rho)+2\\
&= 2\gamma c''(\rho)(\rho_v+\rho_\theta+(k-1)\rho_{\epsilon,k}+\rho)^2+\left[2\gamma c'(\rho)(\rho_v+\rho_\theta+(k-1)\rho_{\epsilon,k}+\rho)-1\right]^2+1\\
&> 0,
\end{aligned}$$

where the inequality follows from the fact that $c''(\cdot)\ge 0$ for a twice continuously differentiable convex function $c(\cdot)$. Thus, $\frac{e^{2\gamma c(\rho)}}{\rho_v+\rho_\theta+(k-1)\rho_{\epsilon,k}+\rho}$ is a convex function of ρ. Moreover, it tends to ∞ as $\rho\to\infty$; hence, the value at which function $\frac{e^{2\gamma c(\rho)}}{\rho_v+\rho_\theta+(k-1)\rho_{\epsilon,k}+\rho}$ achieves its minimum exists and is unique. Consequently, it suffices to show that there exists $\rho_{\epsilon,k}>0$ such that

$$\left.\frac{\partial\left[\frac{e^{2\gamma c(\rho)}}{\rho_v+\rho_\theta+(k-1)\rho_{\epsilon,k}+\rho}\right]}{\partial\rho}\right|_{\rho=\rho_{\epsilon,k}} = 0,$$

i.e., $H(\rho_{\epsilon,k})=0$, or, equivalently, (3) holds with $\rho=\rho_{\epsilon,k}$.

From the condition $c'(0) = 0$ and twice-continuous differentiability of $c(\cdot)$, we see that the left-hand side of (3) is continuous in ρ, with limit zero as $\rho \to 0$, and infinity as $\rho \to \infty$. Hence, there exists some $\rho_{\epsilon,k} > 0$ such that (3) holds by applying the intermediate value theorem. Furthermore, because the function $c'(\cdot)$ is non-decreasing (from the convexity of $c(\cdot)$), it follows from the strict increasingness of $c(\cdot)$ that $c'(\rho) > 0$ for any $\rho > 0$. Hence, the positive solution to (3) and then the symmetric linear equilibrium is unique. This completes the proof. □

Proof of Proposition 2

This follows directly from Eq. (3). □

Proof of Proposition 3

Recall that we have shown in Proposition 2 that $z_k = k\rho_{\epsilon,k}$ increases with k. We first show part (i). From (2), we see that

$$\frac{1}{\mathrm{Var}[v|p]} = \rho_v + \rho_\theta = \rho_v + \frac{z_k^2\rho_x}{\gamma^2},$$

which increases with k. The liquidity is given by

$$\frac{1}{\alpha_x} = \frac{z_k\left(\rho_v + \rho_\theta + z_k\right)}{\gamma\left(\rho_\theta + z_k\right)}.$$

Elementary computation shows that $\partial(1/\alpha_x)/\partial k > 0$ if and only if $\rho_\theta + 2\sqrt{\rho_\theta/\rho_x} > \rho_v/\gamma + 1/\rho_x$. The trading volume is expressed as follows:

$$\begin{aligned}
\mathbb{E}|d^*_{i,g}| &= \sqrt{\frac{2}{\pi}\,\mathrm{Var}(d^*_{i,g})} \\
&= \sqrt{\frac{2}{\pi}}\sqrt{\mathrm{Var}\left(\frac{\mathbb{E}[v-p|\mathcal{F}_{i,g}]}{\gamma\,\mathrm{Var}[v|\mathcal{F}_{i,g}]}\right)} \\
&= \frac{\sqrt{2/\pi}}{\gamma\,\mathrm{Var}[v|\mathcal{F}_{i,g}]}\sqrt{\mathrm{Var}(v-p) - \mathrm{Var}[v|\mathcal{F}_{i,g}]} \\
&= \frac{\sqrt{2/\pi}\left(\rho_v+\rho_\theta+z_k\right)}{\gamma}\sqrt{\frac{\gamma^2\rho_v + (\gamma^2+z_k\rho_x)^2\rho_x^{-1}}{\gamma^2(\rho_v+\rho_\theta+z_k)^2} - \frac{1}{\rho_v+\rho_\theta+z_k}} \\
&= \frac{\sqrt{2/\pi}}{\gamma}\sqrt{z_k + \gamma^2\rho_x^{-1}},
\end{aligned} \tag{A.3}$$

where the first equality follows from the formula that $\mathbb{E}|y| = \sqrt{2\sigma^2/\pi}$ if $y \sim N(0,\sigma^2)$, the second one from the optimal demand strategy $d^*_{i,g} = \frac{\mathbb{E}[v|\mathcal{F}_{i,g}]-p}{\gamma\,\mathrm{Var}[v|\mathcal{F}_{i,g}]}$, the third one from the law of total variance, and the fourth one from (2). Hence, it follows from (A.3) that trading volume increases with k.

We now show part (ii). For simplicity, we consider the certainty equivalent instead of the welfare. We have

$$\begin{aligned}
\mathbb{E}\left[U(d^*_{i,g}(v-p) - c(\rho_{\epsilon,k}))\right] &= -e^{\gamma c(\rho_{\epsilon,k})}\sqrt{\frac{1}{\rho_v+\rho_\theta+k\rho_{\epsilon,k}}\,\frac{1}{(1-\alpha_v)^2\rho_v^{-1}+\alpha_x^2\rho_x^{-1}}} \\
&= -e^{\gamma c(\rho_{\epsilon,k})}\sqrt{1 - \frac{\gamma^4\rho_x^{-1}+\gamma^2 z_k}{\rho_x z_k^2 + 2\gamma^2 z_k + \gamma^2\rho_v + \gamma^4\rho_x^{-1}}},
\end{aligned} \tag{A.4}$$

where the first equality follows from (2) and (A.2), and (A.4) follows from the expressions of α_v and α_x given in Proposition 1. Then, from (A.4), the certainty equivalent equals

$$\begin{aligned}
CE(k) &:= -\frac{1}{\gamma}\log\left[\mathbb{E}\left(\exp\left\{-\gamma d^*_{i,g}(v-p)\right\}\right)\right] - c(\rho_{\epsilon,k}) \\
&= \frac{1}{2\gamma}\log\left[\frac{\rho_x z_k^2 + 2\gamma^2 z_k + \gamma^2\rho_v + \gamma^4\rho_x^{-1}}{\rho_x z_k^2 + \gamma^2 z_k + \gamma^2\rho_v}\right] - c(\rho_{\epsilon,k}).
\end{aligned} \tag{A.5}$$

We complete the proof by showing that $\partial CE(k)/\partial k \propto A_k$ given in the proposition, and A_k strictly decreases with k. From (3) and (A.5), we have

$$\begin{aligned}
CE(k) &= \left[\frac{1}{2\gamma}\log\left(\rho_x z_k^2 + 2\gamma^2 z_k + \gamma^2\rho_v + \gamma^4\rho_x^{-1}\right)\right] - \left[\frac{1}{2\gamma}\log\left(\frac{\gamma}{2c'(\rho_{\epsilon,k})}\right) + c(\rho_{\epsilon,k})\right] \\
&=: F_1(k) - F_2(k).
\end{aligned}$$

Then,

$$\frac{\partial CE(k)}{\partial k} = \frac{\partial F_1(k)}{\partial z_k}\frac{\partial z_k}{\partial k} - \frac{\partial F_2(k)}{\partial \rho_{\epsilon,k}}\frac{\partial \rho_{\epsilon,k}}{\partial k}$$

$$= \frac{1}{2\gamma}\frac{2\rho_x z_k + 2\gamma^2}{\rho_x z_k^2 + 2\gamma^2 z_k + \gamma^2\rho_v + \gamma^4\rho_x^{-1}}\frac{\partial z_k}{\partial k} - \left[-\frac{c''(\rho_{\epsilon,k})}{2\gamma c'(\rho_{\epsilon,k})} + c'(\rho_{\epsilon,k})\right]\frac{\partial\rho_{\epsilon,k}}{\partial k}$$
$$= \frac{1}{2\gamma}\frac{2\rho_x z_k + 2\gamma^2}{\rho_x z_k^2 + 2\gamma^2 z_k + \gamma^2\rho_v + \gamma^4\rho_x^{-1}}\left(\rho_{\epsilon,k} + k\frac{\partial\rho_{\epsilon,k}}{\partial k}\right) - \left[-\frac{c''(\rho_{\epsilon,k})}{2\gamma c'(\rho_{\epsilon,k})} + c'(\rho_{\epsilon,k})\right]\frac{\partial\rho_{\epsilon,k}}{\partial k}. \tag{A.6}$$

Taking the derivative of both sides of (3) with the identity $\rho = \rho_{\epsilon,k}$ with respect to k leads to

$$2\gamma^2\rho_v c''(\rho_{\epsilon,k})\frac{\partial\rho_{\epsilon,k}}{\partial k} + 2\gamma^2 c'(\rho_{\epsilon,k})\rho_{\epsilon,k} + 2\gamma^2 k\left[c''(\rho_{\epsilon,k})\rho_{\epsilon,k} + c'(\rho_{\epsilon,k})\right]\frac{\partial\rho_{\epsilon,k}}{\partial k} + 4c'(\rho_{\epsilon,k})\rho_x k\rho_{\epsilon,k}^2$$
$$+ 2\rho_x k^2\left[c''(\rho_{\epsilon,k})\rho_{\epsilon,k}^2 + 2c'(\rho_{\epsilon,k})\rho_{\epsilon,k}\right]\frac{\partial\rho_{\epsilon,k}}{\partial k} = 0,$$

from which and the assumption $c(\rho) = \xi\rho^2$, we have

$$\frac{\partial\rho_{\epsilon,k}}{\partial k} = -\frac{\gamma^2\rho_{\epsilon,k}^2 + 2\rho_x k\rho_{\epsilon,k}^3}{\gamma^2\rho_v + 2\gamma^2 k\rho_{\epsilon,k} + 3\rho_x k^2\rho_{\epsilon,k}^2}. \tag{A.7}$$

Substituting (A.7) into (A.6) and noting $c(\rho) = \xi\rho^2$ again, we obtain

$$\frac{\partial CE(k)}{\partial k} \propto 4\xi\gamma^4 k\rho_{\epsilon,k}^3 + (2\gamma\rho_x k + 4\xi\gamma^6\rho_x^{-1} - \gamma\rho_x k^2 - 8\xi\gamma^4\rho_v)\rho_{\epsilon,k}^2 + \gamma^3(3-2k)\rho_{\epsilon,k} + \gamma^3\rho_v - \gamma^5\rho_x^{-1}$$
$$=: A(k).$$

Next, we show that $\partial A(k)/\partial k < 0$ for any $k \in [1, N]$. First, we have

$$\frac{\partial A(k)}{\partial k} = \left[12\xi\gamma^4 k\rho_{\epsilon,k}^2 + 4\gamma\rho_x k\rho_{\epsilon,k} + 8\xi\gamma^6\rho_x^{-1}\rho_{\epsilon,k} - 2\gamma\rho_x k^2\rho_v - 16\xi\gamma^4\rho_v\rho_{\epsilon,k} + \gamma^3(3-2k)\right]\frac{\partial\rho_{\epsilon,k}}{\partial k}$$
$$+ 4\xi\gamma^4\rho_{\epsilon,k}^3 + 2\gamma\rho_x\rho_{\epsilon,k}^2 - 2\gamma\rho_x k\rho_{\epsilon,k}^2 - 2\gamma^3\rho_{\epsilon,k}.$$

Substituting the expression of $\partial\rho_{\epsilon,k}/\partial k$ in (A.7) into the preceding equation leads to

$$\frac{\partial A(k)}{\partial k} = -\frac{\gamma^2\rho_{\epsilon,k}^2 + 2\rho_x k\rho_{\epsilon,k}^3}{\gamma^2\rho_v + 2\gamma^2 k\rho_{\epsilon,k} + 3\rho_x k^2\rho_{\epsilon,k}^2}\left[12\xi\gamma^4 k\rho_{\epsilon,k}^2 + 4\gamma\rho_x k\rho_{\epsilon,k} + 8\xi\gamma^6\rho_x^{-1}\rho_{\epsilon,k} - 2\gamma\rho_x k^2\rho_v - 16\xi\gamma^4\rho_v\rho_{\epsilon,k} + \gamma^3(3-2k)\right]$$
$$+ 4\xi\gamma^4\rho_{\epsilon,k}^3 + 2\gamma\rho_x\rho_{\epsilon,k}^2 - 2\gamma\rho_x k\rho_{\epsilon,k}^2 - 2\gamma^3\rho_{\epsilon,k}$$
$$= -\frac{1}{\gamma^2\rho_v + 2\gamma^2 k\rho_{\epsilon,k} + 3\rho_x k^2\rho_{\epsilon,k}^2}\Big[12\xi\gamma^6 k\rho_{\epsilon,k}^4 + 24\xi\gamma^4\rho_x k^2\rho_{\epsilon,k}^5 + 4\gamma^3\rho_x k\rho_{\epsilon,k}^3 + 8\gamma\rho_x^2 k^2\rho_{\epsilon,k}^4 + 8\xi\gamma^8\rho_x^{-1}\rho_{\epsilon,k}^3 + 16\xi\gamma^6 k\rho_{\epsilon,k}^4$$
$$- 2\gamma^3\rho_x k^2\rho_{\epsilon,k}^3 - 4\gamma\rho_x^2 k^3\rho_{\epsilon,k}^4 - 16\xi\gamma^6\rho_v\rho_{\epsilon,k}^3 - 32\xi\gamma^4\rho_x\rho_v k\rho_{\epsilon,k}^4 + 3\gamma^5\rho_{\epsilon,k}^2 - 2\gamma^5 k\rho_{\epsilon,k}^2 + 6\gamma^3\rho_x k\rho_{\epsilon,k}^3 - 4\gamma^3\rho_x k^2\rho_{\epsilon,k}^3\Big]$$
$$+ 4\xi\gamma^4\rho_{\epsilon,k}^3 + 2\gamma\rho_x\rho_{\epsilon,k}^2 - 2\gamma\rho_x k\rho_{\epsilon,k}^2 - 2\gamma^3\rho_{\epsilon,k}$$
$$=: -\frac{1}{\gamma^2\rho_v + 2\gamma^2 k\rho_{\epsilon,k} + 3\rho_x k^2\rho_{\epsilon,k}^2}B(k),$$

where

$$B(k) = 12\xi\gamma^6 k\rho_{\epsilon,k}^4 + 24\xi\gamma^4\rho_x k^2\rho_{\epsilon,k}^5 + 4\gamma^3\rho_x k\rho_{\epsilon,k}^3 + 8\gamma\rho_x^2 k^2\rho_{\epsilon,k}^4 + 8\xi\gamma^8\rho_x^{-1}\rho_{\epsilon,k}^3 + 16\xi\gamma^6 k\rho_{\epsilon,k}^4 - 2\gamma^3\rho_x k^2\rho_{\epsilon,k}^3 - 4\gamma\rho_x^2 k^3\rho_{\epsilon,k}^4$$
$$- 16\xi\gamma^6\rho_v\rho_{\epsilon,k}^3 - 32\xi\gamma^4\rho_x\rho_v k\rho_{\epsilon,k}^4 + 3\gamma^5\rho_{\epsilon,k}^2 - 2\gamma^5 k\rho_{\epsilon,k}^2 + 6\gamma^3\rho_x k\rho_{\epsilon,k}^3 - 4\gamma^3\rho_x k^2\rho_{\epsilon,k}^3 - 4\xi\gamma^6\rho_v\rho_{\epsilon,k}^3 - 2\gamma^3\rho_x\rho_v\rho_{\epsilon,k}^2$$
$$+ 2\gamma^3\rho_x\rho_v k\rho_{\epsilon,k}^2 + 2\gamma^5\rho_v\rho_{\epsilon,k} - 8\xi\gamma^6 k\rho_{\epsilon,k}^4 - 4\gamma^3\rho_x k\rho_{\epsilon,k}^3 + 4\gamma^3\rho_x k^2\rho_{\epsilon,k}^3 + 4\gamma^5 k\rho_{\epsilon,k}^2 - 12\xi\gamma^4\rho_x k^2\rho_{\epsilon,k}^5 - 6\gamma\rho_x^2 k^2\rho_{\epsilon,k}^4$$
$$+ 6\gamma\rho_x^2 k^3\rho_{\epsilon,k}^4 + 6\gamma^3\rho_x k^2\rho_{\epsilon,k}^3.$$

We can further simplify $B(k)$ as follows:

$$B(k) = 12\xi\gamma^4\rho_x k^2\rho_{\epsilon,k}^5 + 20\xi\gamma^6 k\rho_{\epsilon,k}^4 + 2\gamma\rho_x^2 k^3\rho_{\epsilon,k}^4 + 2\gamma\rho_x^2 k^2\rho_{\epsilon,k}^4 - 32\xi\gamma^4\rho_x\rho_v k\rho_{\epsilon,k}^4 + 6\gamma^3\rho_x k\rho_{\epsilon,k}^3 + 4\gamma^3\rho_x k^2\rho_{\epsilon,k}^3 - 20\xi\gamma^6\rho_v\rho_{\epsilon,k}^3$$
$$+ 8\xi\gamma^8\rho_x^{-1}\rho_{\epsilon,k}^3 + 2\gamma^5 k\rho_{\epsilon,k}^2 + 3\gamma^5\rho_{\epsilon,k}^2 + 2\gamma^3\rho_x\rho_v\rho_{\epsilon,k}^2(k-1) + 2\gamma^5\rho_v\rho_{\epsilon,k}. \tag{A.8}$$

Then, from (3), we have

$$-4\xi\gamma^2\rho_v\rho_{\epsilon,k} = 4\xi\gamma^2 k\rho_{\epsilon,k}^2 + 4\xi\rho_x k^2\rho_{\epsilon,k}^3 - \gamma.$$

As a result,

$$-32\xi\gamma^4\rho_v\rho_x k\rho_{\epsilon,k}^4 = 8\gamma^2\rho_x k\rho_{\epsilon,k}^3(-4\xi\gamma^2\rho_v\rho_{\epsilon,k})$$
$$= 8\gamma^2\rho_x k\rho_{\epsilon,k}^3(4\xi\gamma^2 k\rho_{\epsilon,k}^2 + 4\xi\rho_x k^2\rho_{\epsilon,k}^3 - \gamma)$$
$$= 32\xi\gamma^4\rho_x k^2\rho_{\epsilon,k}^5 + 32\xi\gamma^2\rho_x^2 k^3\rho_{\epsilon,k}^6 - 8\gamma^3\rho_x k\rho_{\epsilon,k}^3, \tag{A.9}$$

and

$$
\begin{aligned}
-20\xi\gamma^6\rho_v\rho_{\epsilon,k}^3 &= 5\gamma^4\rho_{\epsilon,k}^2(-4\xi\gamma^2\rho_v\rho_{\epsilon,k}) \\
&= 5\gamma^4\rho_{\epsilon,k}^2(4\xi\gamma^2 k\rho_{\epsilon,k}^2 + 4\xi\rho_x k^2\rho_{\epsilon,k}^3 - \gamma) \\
&= 20\xi\gamma^6 k\rho_{\epsilon,k}^4 + 20\xi\gamma^4\rho_x k^2\rho_{\epsilon,k}^5 - 5\gamma^5\rho_{\epsilon,k}^2. \qquad \text{(A.10)}
\end{aligned}
$$

Substituting (A.9) and (A.10) into (A.8), we have

$$
\begin{aligned}
B(k) &= 32\xi\gamma^2\rho_x^2k^3\rho_{\epsilon,k}^6 + 64\xi\gamma^4\rho_x k^2\rho_{\epsilon,k}^5 + 40\xi\gamma^6 k\rho_{\epsilon,k}^4 + 2\gamma\rho_x^2k^3\rho_{\epsilon,k}^4 + 2\gamma\rho_x^2k^2\rho_{\epsilon,k}^4 + 4\gamma^3\rho_x k^2\rho_{\epsilon,k}^3 - 2\gamma^3\rho_x k\rho_{\epsilon,k}^3 + 8\xi\gamma^8\rho_x^{-1}\rho_{\epsilon,k}^3 \\
&\quad + 2\gamma^5 k\rho_{\epsilon,k}^2 - 2\gamma^5\rho_{\epsilon,k}^2 + 2\gamma^3\rho_x\rho_v\rho_{\epsilon,k}^2(k-1) + 2\gamma^5\rho_v\rho_{\epsilon,k} \\
&= 32\xi\gamma^2\rho_x^2k^3\rho_{\epsilon,k}^6 + 64\xi\gamma^4\rho_x k^2\rho_{\epsilon,k}^5 + 40\xi\gamma^6 k\rho_{\epsilon,k}^4 + 2\gamma\rho_x^2k^3\rho_{\epsilon,k}^4 + 2\gamma\rho_x^2k^2\rho_{\epsilon,k}^4 + 2\gamma^3\rho_x k^2\rho_{\epsilon,k}^3 + 2\gamma^3\rho_x k\rho_{\epsilon,k}^3(k-1) \\
&\quad + 8\xi\gamma^8\rho_x^{-1}\rho_{\epsilon,k}^3 + 2\gamma^5 k\rho_{\epsilon,k}^2(k-1) + 2\gamma^3\rho_x\rho_v\rho_{\epsilon,k}^2(k-1) + 2\gamma^5\rho_v\rho_{\epsilon,k} \\
&> 0.
\end{aligned}
$$

This competes the proof. □

Appendix B. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.najef.2023.102015.

References

Brown, J. R., Ivkovic, Z., Smith, P. A., & Weisbenner, S. (2008). Neighbors matter: Causal community effects and stock market participation. *The Journal of Finance*, *63*(3), 1509–1531.

Colla, P., & Antonio, M. (2010). Information linkages and correlated trading. *The Review of Financial Studies*, *23*(1), 203–246.

Enward, H., Riyanto, Y. E., & Roy, N. (2019). Costly information acquisition, social networks, and asset prices: Experimental evidence. *The Journal of Finance*, *74*(4), 1975–2010.

Feng, L., & Seasholes, M. S. (2004). Correlated trading and location. *The Journal of Finance*, *59*(5), 2117–2144.

Han, B., Tang, Y., & Yang, L. (2016). Public information and uninformed trading: Implications for market liquidity and price efficiency. *Journal of Economic Theory*, *163*, 604–643.

Han, B., & Yang, L. (2013). Social networks, information acquisition, and asset prices. *Management Science*, *59*(6), 1444–1457.

Hellwig, M. (1980). On the aggregation of information in competitive markets. *Journal of Economic Theory*, *22*(3), 477–498.

Hong, H., Kubik, J., & Stein, J. (2004). Social interactions and stock-market participation. *The Journal of Finance*, *59*(1), 137–163.

Hong, H., Kubik, J., & Stein, J. (2005). The neighbor's portfolio: Word-of-mouth effects in the holdings and trades of money managers. *The Journal of Finance*, *60*(6), 2801–2824.

Ivkovic, Z., & Weisbenner, S. (2007). Information diffusion effects in individual investors' common stock purchases: Covet thy neighbors' investment choices. *The Review of Financial Studies*, *20*(4), 1327–1357.

Kuchler, T., & Stroebel, J. (2021). Social finance. *Annual Review of Financial Economics*, *13*, 37–55.

Kyle, A. S. (1989). Informed speculation with imperfect competition. *Review of Economic Studies*, *56*(3), 317–356.

Lou, Y., & Wang, S. (2021). The equivalence of two rational expectations equilibrium economies with different approach to processing neighbors' information. *Mathematical Social Sciences*, *109*, 93–105.

Ozsoylev, H. N., & Walden, J. (2011). Asset pricing in large information networks. *Journal of Economic Theory*, *146*(6), 2252–2280.

Ozsoylev, H. N., & Walden, J. (2014). Investor networks in the stock market. *The Review of Financial Studies*, *27*(5), 1323–1366.

Pool, V. K., Stoffman, N., & Yonker, S. E. (2015). The people in your neighborhood: Social interactions and mutual fund portfolios. *The Journal of Finance*, *70*(6), 2679–2732.

Shiller, R. J., & Pound, J. (1989). Survey evidence on diffusion of investment among institutional investors. *Journal of Economic Behaviour and Organization*, *12*(1), 47–66.

Verrecchia, R. E. (1982). Information acquisition in a noisy rational expectations economy. *Econometrica*, *50*(6), 1415–1430.

Walden, J. (2019). Trading, profits, and volatility in a dynamic information network model. *Review of Economic Studies*, *86*(5), 2248–2283.