How many financial advisers do you need?

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Abstract

We study a rational expectations equilibrium economy populated by investors and their financial advisers. Without knowing market parameters, investors depend on advisers for investment recommendations. Advisers provide suggestions that are in the best interest of their client investors, who then aggregate suggested strategies under bounded rationality constraints of conformism and regret aversion under ambiguity. Our model can explain why investors consult only few advisers despite the abundance of available options. Our analysis further highlights that the quality of advice is more important than the number of advisers consulted, and that it is never optimal to rely on a single financial adviser.

Keywords: Rational expectations equilibrium, investment advice, optimal aggregation

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1 Introduction

Many investors turn to financial advisers to form their investment strategies (Bhattacharya et al. 2012; Foerster et al. 2017; Gennaioli et al. 2015).[1] However, despite the wide array of advisers available, investors rarely consult all possible sources. This behavior is inconsistent with traditional models of fully rational agents in rational expectations equilibrium economies, where investors would optimally incorporate all accessible information. Selective consultation with a small number of financial advisers points to underlying cognitive or resource constraints of investors. How many financial advisers do investors with bounded rationality need? And how to optimally aggregate suggestions from different advisers?

The goal of this paper is to formally study these questions within a rational expectations equilibrium framework. We extend the classical model of Hellwig (1980) by introducing two types of agents: Investors with bounded rationality and their financial advisers. Unlike fully rational agents, these investors lack full financial literacy and do not know the key market parameters needed to construct optimal investment strategies. Instead, they depend on financial advisers who observe private signals about the fundamental of the risky asset and recommend strategies maximizing their client investors' expected utility.

Investors can observe suggested strategies of up to n financial advisers, and doing so is free of charge. Upon receiving suggested investment strategies from their advisers, investors optimally aggregate these suggestions under three constraints of bounded rationality: lack of financial literacy, conformism, and regret aversion under ambiguity. Lack of financial literacy states that investors do not know the true market parameters. Conformism reflects a behavioral bias where investors align their actions with the direction of the advice. They do not short the stock if all advisers recommend a long position, and vice versa. Regret aversion under ambiguity requires that the aggregated strategy must perform at least as well as the strategy suggested by any single adviser. Because the investor does not know market parameters, we require it to hold for any value thereof. While investors are subject to the aforementioned constraints of bounded rationality, we assume in the main model of the paper that they know the quality of advisers

[1] These studies highlight the complexities of financial advice, showing that while advisers play a crucial role in guiding investor decision-making, their effectiveness is often limited by conflicts of interest, behavioral biases, and a lack of personalized solutions.

in terms of their signal precision, which can be inferred, for example, through their reputation and historical record. This assumption is later relaxed in an alternative version of the model.

How many advisers should an investor consult in this context, and how should they optimally aggregate the suggested strategies? Our model provides analytical answers to both questions and explains why investors consult only a few advisers despite the abundance of available options. The reasoning is as follows: Conformism implies that the weight assigned to each suggested strategy is non-negative. Lack of financial literacy and regret aversion under ambiguity imply that investors do not incorporate information present in the price when aggregating suggestions, and that suggested strategies are aggregated as a weighted average with weights summing up to one. We term these implications *directional adherence*, *price information neglect*, and *sum-of-weights-equals-one heuristic*. Without the sum-of-weights-equals-one heuristic, investors would have to incorporate information contained in the price to outperform each individual suggested strategy. However, incorporating price information requires knowing market parameters which investors do not know, and using incorrect market information leads to potentially disastrous outcomes incompatible with regret aversion under ambiguity. Thus, investors prefer to incorporate the correct price information already captured in the strategies suggested by advisers, displaying price information neglect.

As a consequence of price information neglect and the sum-of-weights-equals-one heuristic, optimal aggregation policies disregard some of the suggested strategies and discriminate between the remaining strategies by giving higher weights to strategies suggested by high-quality advisers. This occurs because the negative effect of overcounting price information outweighs the benefit of incorporating information from additional low-quality signals. Consequently, the investor assigns a lower weight to such a recommendation or even disregards it completely. The marginal benefit from increasing the weight given to a suggested strategy is decreasing and goes to zero as the weight given to that strategy goes to one. Therefore, investors should consult at least two advisers. This finding shares a common theme with Baldauf and Mollner (2024), who developed a theoretical model explaining why institutional investors often limit both the number of potential counterparties they engage with and the extent of information they disclose in over-the-counter (OTC) markets.

An important insight from our analysis is that quality trumps quantity when it comes to financial advice for investors with bounded rationality. This suggests a potential policy implication: regulatory bodies should maintain and enhance standards for professional financial advisers and improve transparency to help investors identify the best advisers.

Another key finding of our paper is that the optimal number of advisers is small but greater than one. Even in a model without conflicts of interest, where advisers act in the best interest of their clients, it is not optimal to follow the suggestion of a single financial adviser. This highlights the importance of fostering a competitive landscape for financial advice.

The remainder of this paper is organized as follows. We review the related literature in Section 1.1. In Section 2, we introduce the model of a rational expectations equilibrium economy populated by investors with bounded rationality and their financial advisers. Our main results are in Section 3. Further discussions on modeling assumptions are covered in Section 4. We conclude the paper in Section 5. The Appendix contains all proofs.

1.1 Literature review

Our paper is related to the following three main strands of research. First, our bounded rationality constraint of regret aversion under ambiguity is inspired by the well-established concepts of regret aversion and ambiguity aversion in behavioral economics. Regret aversion refers to the tendency to avoid actions that could lead to future regret, often at the expense of better outcomes. It plays a significant role in shaping individual decision-making (Bell 1982; Loomes and Sugden 1982). This concept can explain why individuals often make suboptimal choices, as they prioritize avoiding potential regret over pursuing potentially better outcomes. The concept of ambiguity aversion dates back to Knight (1921), who distinguishes between risk and uncertainty. Ellsberg (1961) challenges the Savage axioms for subjective expected utility Savage (1954) by demonstrating, through thought experiments, that individuals systematically prefer known risks over unknown probabilities. This empirical pattern is now known as ambiguity aversion. More recently, a strand of the literature on ambiguity aversion has focused on implications for asset prices and market participation (Cao et al. 2011, 2005; Easley and O'Hara 2009, 2010; Epstein and Schneider 2010). In this body of literature, investors face imperfect information regarding specific parameters of the asset payoff distribution, thereby encountering model uncertainty. A common assumption is that certain investors do not know the true mean and variance of asset payoffs, instead operating under the premise that these parameters reside within a set consisting of all possible parameters. Consequently, their decision-making process integrates both risk and ambiguity considerations. Ambiguity-averse investors choose portfolios that maximize their minimum expected utility across the set of plausible distributions, and in particular under worst-case scenarios. This approach helps explain why ambiguity-averse investors may choose not to participate in certain assets or asset classes, offering a theoretical basis for the well-documented nonparticipation puzzle in financial markets. Furthermore, this literature explores regulatory and market design strategies aimed at mitigating ambiguity, thereby enhancing market participation and generating welfare improvements.

In our paper, regret aversion under ambiguity integrates the core principles of regret aversion and ambiguity aversion. Specifically, investors insist that their chosen strategy should perform at least as well as any single adviser's suggested strategy. This constraint reflects a behavioral tendency to avoid future regret, consistent with classical regret aversion—specifically, the regret of encountering a single adviser whose suggested strategy would have outperformed the investor's aggregated strategy. Given that investors do not know the true market parameters, we further require that this constraint must hold for any possible market parameter unknown to the investors, particularly in the worst-case scenario, consistent with ambiguity aversion.

Second, we contribute to the emerging literature on behavioral rational expectations equilibria (REE), which departs from the classical Hellwig (1980) framework by relaxing the assumption of fully rational traders. (Banerjee 2011; Banerjee et al. 2024, 2009; Bastianello and Fontanier 2025; Eyster et al. 2019; Kyle and Wang 1997; Mondria et al. 2022). Kyle and Wang (1997) studied a financial market in which traders may overestimate/underestimate the precision of their own and opponent's signals. They show that an overconfident trader can outperform a rational opponent. This is because overconfidence acts like a commitment device in a standard Cournot duopoly model, leading the overconfident trader to trade more aggressively, while the rational trader responds by trading more cautiously. Mondria et al. (2022) developed a model in which investors cannot process price information in financial markets without incurring a cost. They show that such bounded rationality can generate price momentum, excessive return volatility, and excessive trading volume.

Within the behavioral REE, our work is closely related to studies that posit certain traders disregard price information when inferring fundamental values (Banerjee 2011; Banerjee et al. 2024, 2009; Bastianello and Fontanier 2025; Eyster et al. 2019). While there is no price drift in a rational expectations equilibrium, Banerjee et al. (2009) find that developing models with difference of opinions is necessary to generate price drift as an outcome of slow aggregation of heterogeneous beliefs. Banerjee (2011) developed a dynamic model that nests rational expectations and differences of opinion and explored how disagreement shapes market performance—finding that the relationships reverse when investors do not learn from prices. Eyster et al. (2019) modeled a setting in which certain traders, termed "cursed," fail to fully appreciate what prices convey about others' private information. They demonstrate that this cognitive bias can lead to significantly higher trading volumes compared to markets populated exclusively by fully rational traders. Bastianello and Fontanier (2025) advanced a theory of Partial Equilibrium Thinking (PET), positing that uninformed traders infer fundamental information from prices but fail to realize that other uninformed traders are similarly learning from prices. Their work establishes PET as a micro-foundation for over-reaction to news, wherein uninformed traders exhibit upward-sloping demand curves, thereby contributing to greater market inelasticity. Additionally, they show that mislearning from prices can substantially exacerbate mislearning from fundamentals. Building on evidence that agents derive contemporaneous utility from their beliefs about future events, Banerjee et al. (2024) demonstrated that traders in financial markets endogenously choose to disagree about both private and price information. Specifically, when the objective informativeness of prices is sufficiently low, all traders choose to disregard the information of others. Conversely, when prices are sufficiently informative, the majority of traders dismiss price information, while a minority conditions their decisions on it. Our paper shares with this literature the key feature that investors are boundedly rational. However, a central difference is that, in our model, investors do not know the true market parameters and therefore cannot construct investment strategies on their own. Instead, their main challenge is how to optimally aggregate the strategies recommended by their advisers.

Third, our work is also related to the literature on non-Bayesian learning over networks, such as DeMarzo et al. (2003), Golub and Jackson (2010), Jadbabaie et al. (2012), and Molavi et al. (2018). In this literature, the mechanism of averaging often plays a central role in reducing noise in agents' decisions (Kahneman et al. 2021). Building on the averaging approach proposed by Degroot (1974), DeMarzo et al. (2003) studied a learning model where agents receive independent noisy signals about an unknown parameter. They communicate with their neighbors in a social network, and update their beliefs by repeatedly averaging the opinions of their neighbors. They show that beliefs converge to a consensus belief, which is correct if and only if the network structure is balanced. Following DeMarzo et al. (2003), Golub and Jackson (2010) further show that agents' beliefs can be asymptotically accurate as the network becomes large even if it may not be optimal in finite societies. In contrast to these studies, we do not examine the accuracy of agents' beliefs about an unknown parameter, nor do we exogenously impose a weighted averaging rule. Instead, we consider an exchange economy and focus on how investors optimally aggregate strategy recommendations from advisers. In this context, a weighted average strategy emerges endogenously as a consequence of two behavioral biases of regret aversion under ambiguity and conformism.

2 The economy

Building on the finite-agent noisy rational expectations equilibrium economy of Hellwig (1980), we consider a model featuring two types of agents: Investors with bounded rationality and their fully rational financial advisers. The timeline of our model consists of three dates: date 0, date 1, and date 2. At date 0, advisers construct investment strategies and communicate these strategies to their client investors. At date 1, investors aggregate the strategies suggested by their advisers and submit demand schedules, noise traders trade, and the asset price is endogenously determined. Finally, at date 2, asset payoffs are realized, and investors consume their resulting wealth. The mechanism determining the numerical values of the price and strategies follows the spirit of Hellwig (1980) and Kyle (1989). Specifically, the suggested strategies and the aggregated demand schedule are formulated as functions of the price submitted to an auctioneer. The auctioneer collects demand schedules submitted by all investors, including those of noise traders, calculates a market-clearing price, publicly announces it, and allocates quantities to meet investors' demands accordingly. Investors in our model are of bounded rationality, knowing only attributes of their advisers but not true market parameters.

2.1 Assets and investor utility

We consider an economy where a risk-free asset and a risky asset (stock) are traded by $h \in \mathbb{N}$ investors. The risk-free asset pays zero interest with a perfectly elastic supply, and the risky asset has fundamental value $\theta \sim \mathcal{N}(0, 1/\tau_\theta)$, $\tau_\theta > 0$. To prevent prices from fully revealing, we assume a per-capita random supply u of the risky asset that follows $u \sim \mathcal{N}(0, 1/\tau_u)$, where $\tau_u > 0$ and u is independent of the fundamental value θ. We suppose that preferences of the investors are represented by CARA utility functions and, without further loss of generality, that the wealth of all investors is zero. The utility an investor derives from the (stochastic) terminal wealth $W(x) = x(\theta - p)$ is thus given by

$$U\left(W(x)\right) = -\exp\left(-\rho x(\theta - p)\right),$$

where ρ is the coefficient of risk aversion,[2] x denotes the demand of the investor for the risky asset, and p is the publicly observable price of the risky asset.

2.2 Financial advisers with full rationality

A key feature of our model is that investors do not know the true market parameters of τ_θ and τ_u. This can be because they are not fully financially literate, or because investors are not able to search for information due to an inherent inability to do so or resource constraints.[3] Consequently, investors are unable to construct investment strategies on their own. Instead, investors consult financial advisers, obtain their investment suggestions, and then aggregate these suggestions to derive a strategy. Financial advisers can be traditional wealth managers, market experts, or robo-advisers.

Each investor i can consult up to n advisers, indexed by $(i, 1), (i, 2), \ldots, (i, n)$.[4] We assume

[2]The results of this paper hold for more general cases of heterogeneous risk aversion provided that the limit economy is well-defined. We consider the case of homogeneous risk aversion in order to simplify the setup and notation.

[3]Related, Gennaioli et al. (2015) assume that investors have very limited knowledge of how to invest, are too nervous or anxious to make risky investments on their own, and hence hire money managers and advisers, who can give them confidence to take risks and to help them invest.

[4]We assume that the maximal number of advisers is identical for each investor to simplify notation. Our results also hold when $n = n(i)$ differs across investors. In particular, our model allows for a subset of investors

that investors can communicate their risk preferences to their advisers[5] and that adviser (i, j) provides investor i with an investment suggestion that maximizes the expected utility of investor i. Because each adviser constructs a recommended investment strategy based on their own private signal, a given adviser cannot optimize the investor's expected utility based on the aggregated portfolio.[6]

The advisers of our model do not invest in the market themselves but only provide advice to their clients in terms of suggested investment strategies. Each adviser (i, j), $i = 1, \ldots, h$, $j = 1, \ldots, n$, observes a private signal $y_{ij} = \theta + \epsilon_{ij}$ about the fundamental θ. The idiosyncratic noise $\epsilon_{ij} \sim \mathcal{N}(0, 1/\tau_j)$ is assumed to be unbiased, independent across advisers, and independent of θ and u, where $\tau_j > 0$ denotes the information precision of adviser (i, j) and is assumed to be homogenous across investors and does not depend on i for simplicity.[7] The strategy constructed by adviser (i, j) and communicated to investor i depends on both the private signal and the public price of the risky asset, i.e., $x_{ij} = x_{ij}(y_{ij}, p)$.

We assume that a single adviser's investment strategy does not impact asset prices; thus, advisers do not account for price impact in their decisions. To justify this assumption, we adopt the setting of a large economy by letting the number of investors go to infinity, $h \to \infty$, as in

that are financially literate and construct their own investment strategies by setting $n(i) = 1$ for some $i \in \mathbb{N}$ with the interpretation that adviser and investor coincide in this case. In other words, our results also hold in a more general setting where only a fraction of investors are financially illiterate, and financially literate investors observe private signals and construct their own strategies without consulting financial advisers.

[5]In practice, advisers typically infer the risk preferences of their clients by asking a series of questions designed to elicit risk preferences.

[6]Besides the infeasibility of taking into account other advisers' suggested strategies, there might also be reputational concerns that incentivize advisers to recommend strategies that are in the best interest of their investor when evaluated in isolation.

[7]All results in the paper remain valid for more general cases of heterogeneous signal precision across adviser pools, i.e., when the information precision of adviser (i, j) depends on both i and j. Furthermore, our results also hold under a more general adviser pool structure where advisers can suggest strategies to multiple investors and the adviser pools of any two investors may be different provided that there are infinitely many advisers, finitely many types of signal precisions, and that there is an upper bound on the number of advisers per investor. These assumptions assure that the limiting equilibrium is well defined and takes a linear form of the fundamental θ and the random supply u as in (5). For example, our results also apply to the setting where multiple investors share the same adviser pool. In this case, the number of advisers can be smaller than the number of investors.

Hellwig (1980).

2.3 Investors with bounded rationality

Investors receive strategies suggested by their advisers and then aggregate these strategies to determine a personal investment strategy that maximizes expected utility. A fully rational investor who knows the true values of market parameters and understands how advisers suggest strategies would first infer the signals observed by the advisers and then derive the optimal investment strategy based on all available information. Assuming linearity, the resulting rationally optimal investment strategy would be of the form

$$\sum_{j=1}^{n} a_{ij} x_{ij} + \varphi_i p \tag{1}$$

for some $a_{ij} \geq 0$ and $\varphi_i \in \mathbb{R}$ depending on the signal precision of advisers $(\tau_j)_{j=1,\ldots,n}$ as well as the true market parameters τ_θ and τ_u, see Lemma 1 in the next section for details. If investors were fully rational, the setting in which advisers suggest strategies to investors is essentially equivalent to models of information sharing (Colla and Antonio 2010; Halim et al. 2019; Han and Yang 2013; Lou and Yang 2023; Ozsoylev and Walden 2011; Walden 2019).

However, investors in our model are not fully rational and do not know the true values of the market parameters τ_θ and τ_u. How does an investor with bounded rationality evaluate the performance of an aggregated investment strategy of the form (1) without knowing the true values of market parameters and the strategies of other investors? We assume that investors adopt a subjective belief about the values of the market parameters, denoted as $\widetilde{\tau}_\theta^i$ and $\widetilde{\tau}_u^i$, aware that these may be incorrect, and that investors assume that all advisers also work with these parameters. Additionally, investors recognize that the price is endogenously determined by both random supply and the strategies of other investors, but they do not know the aggregation policies adopted by other investors. Investor i assumes that all other investors adopt an identical but unknown aggregation policy $((\widetilde{a}_j^i)_{j=1,\ldots,n}, \widetilde{\varphi}^i)$. The resulting expected utility from terminal wealth, when adopting an aggregation policy of the form (1), is then given by

$$\mathbb{E}^{\widetilde{\tau}_\theta^i, \widetilde{\tau}_u^i}_{(\widetilde{a}_1^i, \ldots, \widetilde{a}_n^i, \widetilde{\varphi}^i)} \left[U \left(W \left(\sum_{j=1}^{n} a_{ij} x_{ij} + \varphi_i p \right) \right) \right],$$

where $\mathbb{E}^{\widetilde{\tau}^i_\theta,\widetilde{\tau}^i_u}_{(\widetilde{a}^i_1,\dots,\widetilde{a}^i_n,\widetilde{\varphi}^i)}$ denotes the expectation operator in a model where $\theta \sim \mathcal{N}(0, 1/\widetilde{\tau}^i_\theta)$, $u \sim \mathcal{N}(0, 1/\widetilde{\tau}^i_u)$, $\widetilde{\tau}^i_\theta > 0$, $\widetilde{\tau}^i_u > 0$, and p is the price that would emerge when all other investors adopt the same aggregation policy $((\widetilde{a}^i_j)_{j=1,\dots,n}, \widetilde{\varphi}^i)$.

Interestingly, we will later show that the aggregation policy adopted in equilibrium does not depend on individual investors' beliefs about the market parameters $\widetilde{\tau}^i_\theta$, $\widetilde{\tau}^i_u$, and the aggregation profile $((\widetilde{a}^i_j)_{j=1,\dots,n}, \widetilde{\varphi}^i)$ adopted by other investors. Moreover, the assumption that all investors adopt an identical aggregation policy $((\widetilde{a}^i_j)_{j=1,\dots,n}, \widetilde{\varphi}^i)$ indeed holds in equilibrium.

We assume that investors linearly aggregate suggested strategies and the price as in (1), but subject to the following bounded rationality constraints.

Definition 1. *An aggregation policy* $((a_{ij})_{j=1,\dots,n}, \varphi_i) \in \mathbb{R}^n \times \mathbb{R}$ *is called admissible under bounded rationality constraints if*

(i) Lack of financial literacy: the coefficients $(a_{ij})_{j=1,\dots,n}$ *and* φ_i *depend only on the signal precision of their advisers* $(\tau_j)_{j=1,\dots,n}$.

(ii) Conformism: the investor will buy (or sell) the stock whenever all advisers suggest to buy (or sell) the stock.

(iii) Regret aversion under ambiguity: the aggregated strategy (1) must perform at least as well as the suggested strategy of any single adviser for any belief. Formally, for any $\widetilde{\tau}^i_\theta, \widetilde{\tau}^i_u > 0$ *and any aggregation policy* $((\widetilde{a}^i_j)_{j=1,\dots,n}, \widetilde{\varphi}^i) \in \mathbb{R}^n \times \mathbb{R}$ *adopted by all other investors, the following inequality holds:*

$$\mathbb{E}^{\widetilde{\tau}^i_\theta,\widetilde{\tau}^i_u}_{(\widetilde{a}^i_1,\dots,\widetilde{a}^i_n,\widetilde{\varphi}^i)}\left[U\left(W\left(\sum_{j=1}^{n} a_{ij}x_{ij} + \varphi_i p\right)\right)\right] \geq \max_{1\leq j\leq n} \mathbb{E}^{\widetilde{\tau}^i_\theta,\widetilde{\tau}^i_u}_{(\widetilde{a}^i_1,\dots,\widetilde{a}^i_n,\widetilde{\varphi}^i)}\left[U\left(W\left(x_{ij}\right)\right)\right]. \quad (2)$$

The set of aggregation policies admissible under bounded rationality constraints is denoted by $\mathcal{A}$.

The first constraint of bounded rationality states that investors do not know the true market parameters τ_θ and τ_u, which we refer to as *lack of financial literacy*. However, we for now assume that investors know the signal precision of their advisers $(\tau_j)_{j=1,\dots,n}$. The precision of advisers reflects their capabilities and can be proxied, for example, from their reputation and historical performance. A discussion of the case where investors do not know the signal precision of their advisers is postponed to Subsection 4.1. The second constraint of bounded rationality states that, investors exhibit *conformism*: they do not take opposing positions to their advisers. Specifically, investors do not short (long) the stock when all advisers recommend to take a long (short) position.[8] Conformity is an important concept in social psychology, referring to the tendency of individuals to align their beliefs or behaviors with those of the people around them (Bernheim 1994; Cialdini and Goldstein 2004; Deutsch and Gerard 1955).

In behavioral economics, regret aversion refers to the tendency to avoid actions that could lead to future regret (Bell 1982; Loomes and Sugden 1982). This concept can explain why individuals often make suboptimal choices, as they prioritize avoiding potential regret over pursuing potentially better outcomes. Additionally, in the literature on asset prices and market participation under ambiguity aversion (Cao et al. 2011, 2005; Easley and O'Hara 2009, 2010; Epstein and Schneider 2010), investors face imperfect information regarding specific parameters of the asset payoff distribution, thereby integrating both risk and ambiguity considerations into their decision-making. Ambiguity-averse investors select portfolios that maximize their minimum expected utility across the set of plausible distributions. The last constraint of bounded rationality *regret aversion under ambiguity* assumes that investors aim to ensure that their chosen strategy performs at least as well as the strategy suggested by any single adviser. Like classical regret aversion, this constraint characterizes the tendency to avoid the future regret associated with worse outcomes, namely that investors fear being dominated by any single suggested strategy. Because investors do not know the true values of market parameters, this must hold under any value thereof and in particular in the worst case as in ambiguity aversion.

We remark that the set of aggregation policies admissible under bounded rationality constraints $\mathcal{A}$ is nonempty. In fact, all the three constraints in Definition 1 are satisfied when the investor follows the simple heuristic of following the strategy recommended by the adviser with the highest signal precision. Specifically, the aggregation policy defined by $\varphi_i = 0$, $a_{ii_0} = 1$ for $i_0 = \arg\max_{1\leq j\leq n} \tau_j$, and $a_{ij} = 0$ for any $j \neq i_0$, is admissible under bounded rationality constraints.[9] In the main body of this paper, we introduce both constraints of conformism and

[8] All advisers of an investor would recommend to short (long) the stock when the signals advisers observe are simultaneously sufficiently large (small).

[9] However, as demonstrated by our result in Proposition 3, such an aggregation policy is never optimal.

regret aversion under ambiguity. A discussion of how investors optimally aggregate suggested strategies when they exhibit only conformism or regret aversion under ambiguity is postponed to Subsections 4.2 and 4.3.

2.4 Equilibrium

We next introduce the notion of *equilibrium* for the context of our model consisting of investors with bounded rationality and their financial advisers. In contrast to traditional models of rational expectations equilibrium economies, the central challenge lies in understanding how investors aggregate the investment strategies suggested by their advisers.

Definition 2. *An equilibrium is a tuple* $((x_{ij}, a^*_{ij}, \varphi^*_i)_{i=1,\dots,\infty,j=1,\dots,n}, p)$ *of strategies suggested by the advisers, aggregation policies in terms of the coefficients in (1), and the price, such that*

(i) Advisers maximize the expected utility of investors: For each i *and* j*,* x_{ij} *maximizes the expected utility conditional on the private signal* y_{ij} *and price* p*, i.e.,*

$$x_{ij}(y_{ij}, p) \in \arg\max_{x} \mathbb{E}[U\left(W(x)\right)|y_{ij}, p].$$

(ii) Investors optimally aggregate suggested strategies under the constraints of bounded rationality: For each i*, the aggregation policy* $\left((a^*_{ij})_{j=1,\dots,n}, \varphi^*_i\right) \in \mathcal{A}$ *is admissible under bounded rationality constraints and*

$$\left((a^*_{ij})_{j=1,\dots,n}, \varphi^*_i\right) \in \underset{((a_{ij})_{j=1,\dots,n},\varphi_i)\in\mathcal{A}}{\arg\max} \mathbb{E}^{\widetilde{\tau}^i_\theta,\widetilde{\tau}^i_u}_{(\widetilde{a}^i_1,\dots,\widetilde{a}^i_n,\widetilde{\varphi}^i)}\left[U\left(W\left(\sum_{j=1}^{n} a_{ij}x_{ij} + \varphi_i p\right)\right)\right] \tag{3}$$

for any $\widetilde{\tau}^i_\theta, \widetilde{\tau}^i_u > 0$ *and any* $((\widetilde{a}^i_j)_{j=1,\dots,n}, \widetilde{\varphi}^i) \in \mathbb{R}^n \times \mathbb{R}$*.*

(iii) The market clears:

$$\lim_{h\to\infty} \frac{1}{h}\sum_{i=1}^{h}\left(\sum_{j=1}^{n} a^*_{ij}x_{ij}(y_{ij}, p) + \varphi^*_i p\right) = u.$$

Condition (i) states that advisers act in the best interest of their client investors by maximizing their ex-ante expected utility based on their own private signal and the price.[10] Condition

[10] Advisers make recommendations that are in the best interest of investors, for example because of potential benefits to their reputation and influence.

(ii) describes how investors optimally aggregate the suggested strategies communicated to them by their advisers. Because investors do not know market parameters and the aggregation policies adopted by other investors, we require that the adopted aggregation policy is optimal under *any* value thereof. We will later show that such an equilibrium exists and is unique within the family of linear equilibria. Condition (iii) specifies the market-clearing rule, i.e., the demand equals the supply.

In the next section, we characterize equilibria via the following three steps. The first step is to characterize the equilibrium price and the strategies suggested by advisers, assuming that all investors adopt an identical aggregation policy. This follows from standard arguments. The second step is to show that

$$\mathcal{A} \subseteq \left\{ ((a_j)_{j=1,\ldots,n}, \varphi) \,\middle|\, a_j \geq 0, \sum_{j=1}^{n} a_j = 1, \varphi = 0 \right\}.$$

Finally, the third step is to show that there exist unique aggregation policies $(a^*_{ij})_{j=1,\ldots,n}$, which are identical across investors and depend only on the signal precisions of their advisers, such that (3) holds with $\varphi^*_i = \varphi_i = \widetilde{\varphi}^i = 0$. By substituting the optimal weights into the equilibrium strategies and the equilibrium price obtained in the first step, we derive a unique equilibrium.

3 Analysis

This section first analyzes the strategies suggested by advisers in equilibrium, followed by a discussion of how boundedly rational investors optimally aggregate these suggested strategies. To address the second question, we first derive the implications of bounded rationality constraints and then examine the optimal aggregation problem faced by investors under these behavioral implications.

3.1 Advisers' suggested strategies

As in the majority of the literature, we herein focus on *linear* equilibria, i.e., equilibria where strategies suggested by advisers are linear functions of the signal and price, and prices are linear in the signals and per-capita supply. Following the analysis in Hellwig (1980), Ozsoylev and Walden (2011) and Han and Yang (2013), we derive the following convergence result as h

increases to infinity. Due to the homogeneity of risk aversion and signal precisions of advisers across investors, the optimal weights in Condition (ii) of Definition 2 will also be homogeneous. We will thus first assume and later verify that the optimal aggregation policies across investors are the same in equilibrium.

Suppose all investors follow the same aggregation policy given by $((a_j)_{j=1,\dots,n}, \varphi)$. Let $a = \sum_{j=1}^{n} a_j$ and let

$$\Delta = \rho^{-1} \sum_{j=1}^{n} a_j \tau_j \tag{4}$$

be the *risk-adjusted average signal precision* in the economy. As $h \to \infty$, the sequence of equilibrium prices of finite-agent economies converges in probability to[11]

$$p = \frac{1}{\Delta + \frac{a\tau_\theta - \rho\varphi}{a\Delta\tau_u + \rho}} (\Delta\theta - u). \tag{5}$$

In the limit of a large economy, the strategy suggested by adviser (i, j) is equal to

$$x_{ij}(y_{ij}, p) = \frac{\mathbb{E}[\theta|y_{ij}, p] - p}{\rho \operatorname{Var}[\theta|y_{ij}, p]} = \rho^{-1} \left(\tau_j y_{ij} - \left(\tau_j + \frac{\rho(\tau_\theta + \varphi\Delta\tau_u)}{a\Delta\tau_u + \rho} \right) p \right). \tag{6}$$

The first equality in (6) is the standard mean-variance portfolio strategy one obtains under the CARA-normality setting (see, e.g., Equations (6) and (11) in Grossman (1976)). The second equality follows from (5) and the projection theorem for normal random variables.[12]

Additional computations yield the ex-ante expected utility of the suggested strategy x_{ij} under a possibly erroneous belief on market parameters $\widetilde{\tau}_\theta^i$, $\widetilde{\tau}_u^i > 0$ and aggregation policy

[11] We assume $\Delta + \frac{a\tau_\theta - \rho\varphi}{a\Delta\tau_u + \rho} \neq 0$, as otherwise no equilibrium exists in the limit economy as $h \to \infty$. Intuitively, if $\Delta + \frac{a\tau_\theta - \rho\varphi}{a\Delta\tau_u + \rho} = 0$, the aggregated strategy becomes insensitive to the price p, resulting in an equilibrium price with infinite variance and price that converges to positive or negative infinity. Indeed, under the bounded rationality constraints specified in Definition 1, it holds that $a = 1$ and $\varphi = 0$, ensuring that $\Delta + \frac{a\tau_\theta - \rho\varphi}{a\Delta\tau_u + \rho} = \Delta + \frac{\tau_\theta}{\Delta\tau_u + \rho} \neq 0$; see Proposition 1. Additionally, recall that we assumed all random variables have mean zero for notational simplicity. Consequently, the price function p does not include an intercept term.

[12] Throughout the main body of this paper, we assume that advisers are fully rational. A discussion of the case where advisers "*agree to disagree*," disregard the information contained in prices, and infer the fundamental value solely based on their private signals rather than prices is postponed to Subsection 4.4.

$((\widetilde{a}^i_j)_{j=1,\dots,n}, \widetilde{\varphi}^i) \in \mathbb{R}^n \times \mathbb{R}$ adopted by all other investors:[13]

$$
\begin{aligned}
\mathbb{E}^{\widetilde{\tau}^i_\theta, \widetilde{\tau}^i_u}_{(\widetilde{a}^i_1,\dots,\widetilde{a}^i_n,\widetilde{\varphi}^i)}[U(W(x_{ij}))] &= \mathbb{E}^{\widetilde{\tau}^i_\theta, \widetilde{\tau}^i_u}_{(\widetilde{a}^i_1,\dots,\widetilde{a}^i_n,\widetilde{\varphi}^i)}[-\exp(-\rho x_{ij}(y_{ij},p)(\theta - p))] \\
&= -\sqrt{\frac{\mathrm{Var}^{\widetilde{\tau}^i_\theta, \widetilde{\tau}^i_u}_{(\widetilde{a}^i_1,\dots,\widetilde{a}^i_n,\widetilde{\varphi}^i)}[\theta | y_{ij}, p]}{\mathrm{Var}^{\widetilde{\tau}^i_\theta, \widetilde{\tau}^i_u}_{(\widetilde{a}^i_1,\dots,\widetilde{a}^i_n,\widetilde{\varphi}^i)}(\theta - p)}} \\
&= -\left(\mathrm{Var}^{\widetilde{\tau}^i_\theta, \widetilde{\tau}^i_u}_{(\widetilde{a}^i_1,\dots,\widetilde{a}^i_n,\widetilde{\varphi}^i)}(\theta - p)(\widetilde{\tau}^i_\theta + (\widetilde{\Delta}^i)^2 \widetilde{\tau}^i_u + \tau_j)\right)^{-\frac{1}{2}},
\end{aligned} \tag{7}
$$

where $\widetilde{\Delta}^i = \rho^{-1} \sum_{j=1}^n \widetilde{a}^i_j \tau_j$, and $\mathrm{Var}^{\widetilde{\tau}^i_\theta, \widetilde{\tau}^i_u}_{(\widetilde{a}^i_1,\dots,\widetilde{a}^i_n,\widetilde{\varphi}^i)}$ represents the variance operator under investor i's belief about the market parameters $\widetilde{\tau}^i_\theta$, $\widetilde{\tau}^i_u$ and the aggregation policy $((\widetilde{a}^i_j)_{j=1,\dots,n}, \widetilde{\varphi}^i)$ adopted by all other investors. The explanations before Definition 1 on how to calculate $\mathbb{E}^{\widetilde{\tau}^i_\theta, \widetilde{\tau}^i_u}_{(\widetilde{a}^i_1,\dots,\widetilde{a}^i_n,\widetilde{\varphi}^i)}$ apply here as well.

3.2 Aggregation policies under bounded rationality constraints

The following proposition shows how the bounded rationality constraints (Definition 1) translate to formal constraints on the coefficients of aggregation policies.

Proposition 1. *The bounded rationality constraints on aggregation policies* $((a_{ij})_{j=1,\dots,n}, \varphi_i) \in \mathbb{R}^n \times \mathbb{R}$ *have the following implications:*

1. *Conformism implies* $a_{ij} \geq 0$.
2. *Lack of financial literacy together with regret aversion under ambiguity imply* $\varphi_i = 0$ *and* $\sum_{j=1}^n a_{ij} = 1$.
3. *Together, lack of financial literacy, conformism, and regret aversion under ambiguity imply*

$$
\mathcal{A} \subseteq \left\{ ((a_j)_{j=1,\dots,n}, \varphi) \,\middle|\, a_j \geq 0, \sum_{j=1}^n a_j = 1, \varphi = 0 \right\}.
$$

Proposition 1 shows that bounded rationality constraints have three behavioral implications on admissible aggregation policies: conformism implies *directional adherence* to suggested strategies, i.e., $a_{ij} \geq 0$; lack of financial literacy and regret aversion under ambiguity imply

[13] See also the proof of Lemma 2 in Rahi and Zigrand (2018).

the *sum-of-weights-equals-one heuristic*, i.e., $\sum_{j=1}^{n} a_{ij} = 1$, and *price information neglect*, i.e., $\varphi_i = 0$. To better understand these behavioral implications, it is helpful to consider a fully rational investor who infers the signals y_{ij} from the suggested strategies x_{ij}.

LEMMA **1.** *The investment strategy of a fully rational investor, who can infer the signals y_{ij} from the suggested strategies x_{ij}, $j = 1, \ldots, n$, is given by*

$$\frac{\mathbb{E}[\theta|y_{i1}, ..., y_{in}, p] - p}{\rho \operatorname{Var}[\theta|y_{i1}, ..., y_{in}, p]} = \rho^{-1} \left(\sum_{j=1}^{n} \tau_j y_{ij} - \left(\sum_{j=1}^{n} \tau_j + \frac{\rho(\tau_\theta + \varphi \Delta \tau_u)}{a \Delta \tau_u + \rho} \right) p \right). \tag{8}$$

Using (6), (8), and the relation

$$\sum_{j=1}^{n} a_{ij} x_{ij} + \varphi_i p = \frac{\mathbb{E}[\theta|y_{i1}, ..., y_{in}, p] - p}{\rho \operatorname{Var}[\theta|y_{i1}, ..., y_{in}, p]},$$

we can infer that the aggregation policy of a fully rational investor satisfies $a_{ij} = 1$ for all j and $\varphi_i = (n-1)(\tau_\theta + \varphi\Delta\tau_u)/(a\Delta\tau_u + \rho)$ in (1). A fully rational investor gives unit weight to each of the suggested strategies in order to optimally utilize the information contained in the signals. In particular, a fully rational investor increases his position by one if the suggested strategy of any single adviser x_{ij} goes up by one. However, in the presence of multiple advisers ($n \geq 2$), this behavior could potentially overcount the impact of the common component of the fundamental value reflected in the price. To correct for this bias, the fully rational investor adjusts the price using a loading coefficient $\varphi_i = (n-1)(\tau_\theta + \varphi\Delta\tau_u)/(a\Delta\tau_u + \rho)$.

Note that the aggregation policy adopted by a fully rational investor depends on the true market parameters τ_θ and τ_u. This policy can thus not be adopted by the investors with bounded rationality in our model, whose aggregation policies can only depend on the signal precision of their advisers (Constraint (i) in Definition 1). Directional adherence to suggested strategies is a direct consequence of Constraint (ii) in Definition 1, which states that the investor buys (sells) the stock whenever all advisers suggest to buy (sell) the stock. Next, recall that regret aversion under ambiguity (Constraint (iii) in Definition 1) requires that the aggregated strategy must perform at least as well as the suggested strategy of any single adviser for any value of the market parameters $\widetilde{\tau}_\theta^i$ and $\widetilde{\tau}_u^i$. Together with the requirement that aggregation policies are independent of market parameters, this leads to the sum-of-weights-equals-one heuristic and price information neglect. To see this, note that the market parameters enter each adviser's suggested strategy (6) through the common factor $-\frac{\tau_\theta+\varphi\Delta\tau_u}{a\Delta\tau_u+\rho}$ multiplied to the price p. Without the sum-of-weight-equals-one heuristic (i.e., when $\sum_{j=1}^n a_{ij} \neq 1$), the aggregated strategy $\sum_{j=1}^n a_{ij}x_{ij}$ would require an additional price-dependent adjustment term, $\varphi_i p$, to avoid disastrous outcomes under certain market parameters and, ultimately, to ensure that it cannot be dominated by any single suggested strategy across all possible market parameters. The correct adjustment requires knowing market parameters, which investors with bounded rationality do not have. Therefore, investors are compelled to adopt the sum-of-weight-equals-one heuristic. Moreover, since the investor follows this heuristic and each suggested strategy assigns the same weight to the price, it is optimal for investors with regret aversion under ambiguity to display price information neglect. This is because the aggregated strategy $\sum_{j=1}^n a_{ij}x_{ij}$ already contains the same market parameter dependent factor applied to the price as any single suggested strategy does. Intuitively, under the sum-of-weights-equals-one heuristic, an additional adjustment (independent of the true market parameters) with a positive value of φ_i indicates that the investor increases his long position as stock prices increase. Conversely, a negative value of φ_i suggests that the investor takes a significant short position even when the excess return is high, such as in scenarios with low noise and a high fundamental value. These two behaviors can result in a low expected utility and consequently underperforms the suggested strategy of any single adviser.

The sum-of-weights-equals-one heuristic and price information neglect are not unique to our paper. The sum-of-weights-equals-one heuristic has been widely used in the social learning literature, for example, Degroot (1974), DeMarzo et al. (2003), Golub and Jackson (2010), Golub and Jackson (2012), and Jadbabaie et al. (2012). Price information neglect has been assumed in Eyster et al. (2019) to explain trading volume, where traders do not perceive the information content of others' behavior and neglect disagreements in traders' beliefs. In our model, these features of admissible aggregation policies emerge endogenously as a consequence of underlying behavioral assumptions: lack of financial literacy, conformism and regret aversion under ambiguity.

We summarize the conclusions of Proposition 1 for the context of an equilibrium as follows. Investors with bounded rationality aggregate suggested strategies under the following three behavioral patterns: directional adherence, the sum-of-weights-equals-one heuristic and price information neglect. Specifically, after observing the strategies suggested by his advisers $(x_{ij})_{j=1,\dots,n}$, investor i decides on the weights $a_{ij} \geq 0$ satisfying the sum-of-weight-equals-one constraint $\sum_{j=1}^{n} a_{ij} = 1$ and then aggregates the suggested strategies to

$$x_i^* := \sum_{j=1}^{n} a_{ij} x_{ij}.$$

The weights a_{ij} are chosen to maximize investor i's expected utility from the weighted strategy x_i^*, given their belief about the market parameters $\widetilde{\tau}_\theta^i$ and $\widetilde{\tau}_u^i > 0$, and the aggregation policy $((\widetilde{a}_j^i)_{j=1,\dots,n}, \widetilde{\varphi}^i) \in \mathbb{R}^n \times \mathbb{R}$ adopted by all other investors as specified in (3).

We remark that some financial advisers do not only provide investment suggestions but also directly implement investment strategies on behalf of their clients. This is in particular the case for the emerging industry of robo-advisers, see, e.g., D'Acunto et al. (2019), D'Acunto and Rossi (2021), Dai et al. (2021), Capponi et al. (2022), and Liang et al. (2023) for a recent literature discussing the interaction between robo-advisers and their human clients. This setting can also be covered by our model by interpreting $a_{ij} x_{ij}$ as the amount investor i transfers to adviser (i, j) which is then invested in the risky asset by the adviser on behalf of the investor.

3.3 Optimal aggregation of suggested strategies

This section is concerned with the optimal aggregation of strategies suggested by advisers under the constraints of bounded rationality. We first characterize the ex-ante expected utility of the aggregated strategy adopted by investors.

Proposition 2. *For any weight $(a_{ij})_{j=1,\dots,n}$ with $a_{ij} \geq 0$ and $\sum_{j=1}^{n} a_{ij} = 1$ and any belief $\widetilde{\tau}_\theta^i$, $\widetilde{\tau}_u^i > 0$ and $((\widetilde{a}_j^i)_{j=1,\dots,n}, \widetilde{\varphi}^i) \in \mathbb{R}^n \times \mathbb{R}$, the (ex-ante) expected utility of the weighted average strategy $x_i^* = \sum_{j=1}^{n} a_{ij} x_{ij}$ is given by*

$$\begin{aligned}
\mathbb{E}^{\widetilde{\tau}_\theta^i, \widetilde{\tau}_u^i}_{(\widetilde{a}_1^i, \dots, \widetilde{a}_n^i, \widetilde{\varphi}^i)}[U(W(x_i^*))] &= -\left(\operatorname{Var}^{\widetilde{\tau}_\theta^i, \widetilde{\tau}_u^i}_{(\widetilde{a}_1^i, \dots, \widetilde{a}_n^i, \widetilde{\varphi}^i)}(\theta - p) \left(\frac{1}{\operatorname{Var}^{\widetilde{\tau}_\theta^i, \widetilde{\tau}_u^i}_{(\widetilde{a}_1^i, \dots, \widetilde{a}_n^i, \widetilde{\varphi}^i)}[\theta | p]} + \tau_i^E \right) \right)^{-\frac{1}{2}} \\
&= -\left(\operatorname{Var}^{\widetilde{\tau}_\theta^i, \widetilde{\tau}_u^i}_{(\widetilde{a}_1^i, \dots, \widetilde{a}_n^i, \widetilde{\varphi}^i)}(\theta - p) \left(\widetilde{\tau}_\theta^i + (\widetilde{\Delta}^i)^2 \widetilde{\tau}_u^i + \tau_i^E \right) \right)^{-\frac{1}{2}} \\
&= -\left(1 + \rho \alpha_i \beta_i + \tau_i^E \gamma_i \right)^{-\frac{1}{2}},
\end{aligned} \tag{9}$$

where

$$\tau_i^E := \sum_{j=1}^{n} (2a_{ij} - a_{ij}^2)\tau_j, \tag{10}$$

and α_i, β_i *and* γ_i *are given by (19) in the Appendix, depending only on the parameters* $\widetilde{\tau}_\theta^i$, $\widetilde{\tau}_u^i$, ρ *and* $\widetilde{\Delta}^i$, *and independent of the weights* $(a_{ij})_{j=1,\ldots,n}$.

Considering (7), (9) and (10), we find that the expected utility resulting from x_i^* remains the same when replacing the signal precision τ_j of each adviser (i,j) with τ_i^E. We refer to τ_i^E as the *equivalent signal precision*. While the strategy suggested by adviser (i,j) enters the aggregated strategy with weight a_{ij}, the signal precision of adviser (i,j) contributes to the equivalent signal precision with a weight $(2a_{ij} - a_{ij}^2)$. This term is increasing in a_{ij} since $a_{ij} \in [0,1]$ due to directional adherence and the sum-of-weights-equals-one heuristic. Moreover, the higher the signal precision of the advisers, the higher the equivalent signal precision τ_i^E. Remarkably, the equivalent signal precision τ_i^E depends only on the weights $(a_{ij})_{j=1,\ldots,n}$ and the signal precisions $(\tau_j)_{j=1,\ldots,n}$ of advisers. It is entirely independent of other model parameters, including the investor's beliefs about market parameters $\widetilde{\tau}_\theta^i$ and $\widetilde{\tau}_u^i$, and the aggregation policy $((\widetilde{a}_j^i)_{j=1,\ldots,n}, \widetilde{\varphi}^i)$ adopted by other investors.

Since we consider the limit of a large economy, the decision of any individual investor has no impact on the price p and the risk-adjusted average signal precision $\widetilde{\Delta}^i$. These quantities are endogenously determined in equilibrium through the collective decisions of all investors. This observation, together with (9) and (10), implies that the optimal aggregation of suggested strategies can be achieved by maximizing the equivalent signal precision τ_i^E. Specifically, investors maximize the expected utility of their weighted strategies by choosing $(a_{ij}^*)_{j=1,\ldots,n}$ which solves the following maximization problem:

$$\begin{aligned} \sup_{(a_{ij})_{j=1,\ldots,n}} \quad & \sum_{j=1}^{n} (2a_{ij} - a_{ij}^2)\tau_j \\ \text{s.t.} & \sum_{j=1}^{n} a_{ij} = 1, \; a_{ij} \geq 0. \end{aligned} \tag{11}$$

Next, we show how investors optimally aggregate the strategies suggested by their advisers.

PROPOSITION **3.** *Assume without loss of generality that* $\tau_1 \geq \tau_2 \geq \cdots \geq \tau_n$. *The following hold.*

(i) The optimal solution to the optimization problem (11) exists, is unique, contains at least two positive components, and is given by

$$a_{ij}^* = \frac{a_{it}^*\tau_t + \tau_j - \tau_t}{\tau_j} = 1 - \frac{(t-1)\frac{\tau_t}{\tau_j}}{1+\sum_{\ell=1}^{t-1}\frac{\tau_t}{\tau_\ell}},\ j = 1, ..., t-1;$$

$$a_{it}^* = \frac{1+\sum_{\ell=1}^{t-1}\frac{\tau_t-\tau_\ell}{\tau_\ell}}{1+\sum_{\ell=1}^{t-1}\frac{\tau_t}{\tau_\ell}} = 1 - \frac{t-1}{1+\sum_{\ell=1}^{t-1}\frac{\tau_t}{\tau_\ell}};$$

$$a_{ij}^* = 0,\ j = t+1, ..., n,$$

where

$$t = \max\left\{ j \,\middle|\, 1 \le j \le n, 1 + \sum_{\ell=1}^{j-1}\frac{\tau_j}{\tau_\ell} > j-1 \right\} \ge 2.$$

The optimal solution satisfies $a_{i1}^* \ge a_{i2}^* \ge \cdots \ge a_{it}^* > 0$, and the inequality becomes an equality if and only if the corresponding two signal precisions are identical. In particular, when $\tau_1 = \tau_2 = \cdots = \tau_n$, it holds that $a_{i1}^* = a_{i2}^* = \cdots = a_{in}^* = 1/n$.

(ii) The optimal value $\sum_{j=1}^n (2a_{ij}^*(\boldsymbol{\tau}_i) - (a_{ij}^*(\boldsymbol{\tau}))^2)\tau_j$ of (11) is increasing in τ_j for any j, where $\boldsymbol{\tau} = (\tau_1, ..., \tau_n)$. Moreover, if $a_{ij}^* > 0$, it is strictly increasing in τ_j.

We proceed with a discussion of the optimal aggregation policy in equilibrium. We first observe that the optimal aggregation policy adopted by investors includes at least two positive components. This is not an immediate consequence of the regret aversion under ambiguity constraint. For example, simply following the strategy recommended by the adviser with the highest signal precision is admissible but not optimal. This finding suggests that investors should consult at least two advisers, even if the signal precision of the best adviser is significantly higher than that of the second-best adviser. Specifically, in the special case of $n = 2$, we get the explicit solution $a_{ij}^* = \tau_j/(\tau_1+\tau_2)$, $j = 1, 2$. This result indicates that an investor should always consult both advisers, regardless of the disparity in their signal precisions, and that the optimal weights are in proportion to the advisers' signal precisions. The marginal benefit of increasing the weight assigned to a suggested strategy converges to zero as the weight approaches one, but to a positive number if the weight is less than one.[14] Compared to the scenario where all

[14]This can be demonstrated by analyzing the objective function component $(2a_{ij} - a_{ij}^2)\tau_j$ in (11), which is strictly increasing in $a_{ij} \in (0, 1]$ and reaches its maximum at $a_{ij} = 1$ with a derivative of zero.

weight is placed on the strategy suggested by the adviser with the highest signal precision, an investor can increase his expected utility by slightly reducing this weight and correspondingly increasing the weight assigned to another adviser's strategy.

We now provide further intuition on why the optimal aggregation weight must contain at least two positive components. To illustrate this, we first examine the special case where the number of strategies is two ($n = 2$) and the precision parameters are equal ($\tau_1 = \tau_2$). In this scenario, the optimal aggregation weight is $(\frac{1}{2}, \frac{1}{2})$, and the expected utility of the aggregated strategy $\frac{x_{i1}+x_{i2}}{2}$ is given by $\mathbb{E}^{\widetilde{\tau}_\theta^i, \widetilde{\tau}_u^i}_{(\widetilde{a}_1^i, \widetilde{a}_2^i, \widetilde{\varphi}^i)}\left[-\exp\{-\rho\frac{x_{i1}+x_{i2}}{2}(\theta - p)\}\right]$. Given the identical signal precision, the expected utilities associated with each single strategy are necessarily equal. In some states of the world, the realized (ex post) wealth $\frac{x_{i1}+x_{i2}}{2}(\theta - p)$ exceeds $x_{i1}(\theta - p)$, while in other states, it falls below $x_{i1}(\theta - p)$. These two events occur with equal probability because of the homogeneity assumption. If investors were risk-neutral, they would be indifferent between the aggregated strategy $\frac{x_{i1}+x_{i2}}{2}$ and x_{i1}, as both strategies yield identical expected wealth. However, in our model, investors are risk averse and thus prefer the aggregated strategy $\frac{x_{i1}+x_{i2}}{2}$, as it exhibits lower volatility than x_{i1} due to the independence of the noise in the signals y_{i1} and y_{i2}. When $\tau_1 > \tau_2$, the optimal aggregation weight becomes $(\frac{\tau_1}{\tau_1+\tau_2}, \frac{\tau_2}{\tau_1+\tau_2})$, and the probabilities of the two aforementioned events are no longer equal. In this case, the positive effects arising from the first event where the realized wealth $\frac{\tau_1 x_{i1}+\tau_2 x_{i2}}{\tau_1+\tau_2}(\theta - p)$ exceeds $x_{i1}(\theta - p)$ and the reduction in uncertainty due to the independence of signal noises outweigh the negative effect of the second event where the realized wealth $\frac{\tau_1 x_{i1}+\tau_2 x_{i2}}{\tau_1+\tau_2}(\theta - p)$ falls below $x_{i1}(\theta - p)$. Consequently, investor i exhibits a preference for the aggregated strategy $\frac{\tau_1 x_{i1}+\tau_2 x_{i2}}{\tau_1+\tau_2}$ over the single strategy x_{i1}, which generates higher expected utility than x_{i2} due to the precision dominance ($\tau_1 > \tau_2$). For the general case where $n \geq 3$ and signal precisions vary across advisers, the preceding analysis remains applicable. Specifically, by examining the pair of strategies with the highest and second-highest precision levels (i.e., τ_1 and τ_2), we can demonstrate that the optimal solution necessarily contains at least two strictly positive components.

A second observation is that the optimal weights given to the strategies suggested by advisers with comparatively low signal precision are zero. In other words, investors cannot always benefit from more consultations even if additional consultation is free.[15] Consulting with an additional

[15]This contrasts with the setting where agents learn about an unknown parameter by observing several

adviser is only beneficial if their signal precision is in a similar range or higher than that of the advisers already consulted. In fact, for any signal precision $(\tau_j)_{j=1,\cdots,n}$, there exists a threshold $\hat{\tau}$ such that the optimal weight $a^*_{ij} > 0$ if and only if $\tau_j \geq \hat{\tau}$, and $a^*_{ij} = 0$ if and only if $\tau_j < \hat{\tau}$.

To better understand why it is not always beneficial to incorporate a suggestion from an additional adviser we recall from Proposition 1 that investors with bounded rationality display directional adherence, the sum-of-weights-equals-one heuristic, and price information neglect. As discussed in Subsection 3.2, an aggregated strategy $\sum_{j=1}^{n} a_{ij}x_{ij}$ may lead to overcounting the fundamental component reflected in the price, which is common to all suggested strategies. To address this, consider an optimal aggregation of strategies from n' $(1 \leq n' < n)$ advisers. We now examine how investors optimally integrate an additional $(n'+1)$-th strategy into their existing optimal aggregation, given the precision ordering $\tau_1 > \tau_2 > \cdots > \tau_n$. When $n' = 1$, the suggested strategy x_{i1} optimally incorporates price information without overcounting information on the fundamental reflected in the price. For relatively small values of n', the negative overcounting effect remains modest, allowing the informational benefit from the additional strategy to dominate the associated overcounting costs. This leads to a positive weight assignment for the additional $(n'+1)$-th strategy. However, when n' becomes large, the additional adviser's signal precision becomes relatively low, but the overcounting effect of price information persists at the same level.[16] The cost of overcounting the impact of the price p may outweigh the benefit of the extra information content carried by the additional suggested strategy. Subject to both the non-negativity constraint and the sum-of-weights-equals-one constraint, it is thus optimal for the investor to exclude this additional suggested strategy from their aggregation.

Third, we observe that $t = n$ exactly when $\tau_n > \frac{n-2}{n-1}\frac{n-1}{\sum_{\ell=1}^{n-1}\frac{1}{\tau_\ell}}$, a multiple of the harmonic mean of the $(n-1)$ higher signal precisions. Therefore, a given investor i should consult all of his advisers if and only if the difference between the highest precision and the other

conditionally independent signals. In such a setting, agents always put a positive weight on each signal, even with very low precision (see the example on page 378 in Vives (2008)). Any additional signal, even with very low precision, can always improve the estimate. However, this is not the case in our optimal aggregation problem because there is the issue of overcounting the price.

[16] This is because the overcounting term $-\frac{\tau_\theta}{\Delta\tau_u+\rho}p$ in (12) does not vary with any single adviser's signal precision.

precisions is small. In particular, when $\tau_1 = \tau_2 = \cdots = \tau_{n-1}$, $t = n$ if and only if $\tau_n > \frac{n-2}{n-1}\tau_{n-1}$. For a^*_{in} to be positive, τ_n needs to be close to τ_{n-1}, especially when n is large. This is because the marginal benefit from increasing the weight given to a suggested strategy is decreasing in the weight that has already been given. If the precision of one additional signal is not too low compared with those the investor has already consulted, the benefit of the extra information conveyed by this signal outweighs the cost of overcounting the price. Consequently, the additional suggested strategy with this signal will receive a positive weight. We remark that when the signal precisions are identical, i.e., $\tau_1 = \cdots = \tau_n$, the optimal weight is uniform: $a^*_{i1} = \cdots = a^*_{in} = \frac{1}{n}$. This follows because, in this case, (11) simplifies to the optimization problem $\max_{(a_{ij})_{j=1,\ldots,n}} \sum_{j=1}^n (2a_{ij} - a_{ij}^2)$, subject to the constraints $\sum_{j=1}^n a_{ij} = 1$ and $a_{ij} \geq 0$. Given the strict concavity of the objective function, any convex combination of a feasible solution and $(\frac{1}{n}, \cdots, \frac{1}{n})$ remains feasible and attains a higher objective value. Consequently, the optimal solution must be $(\frac{1}{n}, \cdots, \frac{1}{n})$.

Fourth, when $\tau_2 = \tau_3 = \cdots = \tau_n$, $t = n$ for any $\tau_1 > \tau_n$. That is, if there is one star adviser and all other advisers share the same lower signal precision, investors should consult all advisers no matter how large the difference in precision between the star adviser and all others. In this case,

$$a^*_{i1} = \frac{(n-1)(\tau_1 - \tau_n) + \tau_n}{(n-1)\tau_1 + \tau_n}, \quad a^*_{i2} = \cdots = a^*_{in} = \frac{\tau_n}{(n-1)\tau_1 + \tau_n}.$$

The optimal aggregation $x^*_i = \sum_{j=1}^n a_{ij} x_{ij}$ can be written as

$$x^*_i = a^*_{i1} x_{i1} + (1 - a^*_{i1}) \frac{\sum_{j=2}^n x_{ij}}{n-1},$$

which is a weighted average of two strategies: x_{i1} and $\frac{\sum_{j=2}^n x_{ij}}{n-1}$. It follows from (12) that the average strategy

$$\frac{\sum_{j=2}^n x_{ij}}{n-1} = \rho^{-1} \left(\tau_n \left(\theta + \frac{\sum_{j=2}^n \epsilon_{ij}}{n-1} \right) - \left(\tau_n + \frac{\rho\tau_\theta}{\Delta\tau_u + \rho} \right) p \right).$$

The aggregated strategy can be interpreted as a hypothetical strategy proposed by a single adviser with private signal $\theta + \frac{\sum_{j=2}^n \epsilon_{ij}}{n-1}$ and precision $(n-1)\tau_n$. However, this strategy is assigned an effective lower weight of τ_n. Intuitively, while the positive informational content of the average strategy is underestimated (i.e., $\tau_n < (n-1)\tau_n$), its negative overcounting effect is relatively amplified. To mitigate the overcounting effect, the average strategy is assigned a weight $1 - a^*_{i1} = \frac{(n-1)\tau_n}{(n-1)\tau_1+\tau_n}$, which is strictly lower than the theoretically optimal weight $\frac{(n-1)\tau_n}{\tau_1+(n-1)\tau_n}$ (due to $\tau_1 > \tau_n$). The latter represents the optimal aggregation weight that would be assigned when aggregating two strategies with precision parameters τ_1 and $(n-1)\tau_n$, as demonstrated in our earlier analysis of the special case where $n = 2$.

Fifth, we observe that for the positive components of the optimal solution, a higher signal precision corresponds to a larger optimal weight. Indeed, if a strategy with higher precision were assigned a lower optimal weight, interchanging its weight with that of a strategy with lower precision would result in a higher value of the objective function.[17] Intuitively, swapping such two weights can enhance the informational content of the aggregated strategy without amplifying the overcounting effect. This is because the overcounting effect, captured by the term $-\frac{\tau_\theta}{\Delta\tau_u+\rho}p$ in (6), remains invariant with respect to advisers. Consequently, such a reallocation improves the expected utility of the aggregated strategy.

Finally, Part (ii) of Proposition 3 shows the intuitive result that the expected utility of an investor increases in the precision of the advisers. Consider the situation where advisers are relatively homogeneous such that the investor consults all of them. Increasing the precision of a given adviser might lead to a situation where the investor disregards some of the suggestions. The proposition shows that, in this scenario, the resulting welfare of the investor, after the increase in the precision of a single adviser and the corresponding adjustment of the optimal aggregation weight, is still superior to the original setting where the investor consults all advisers.

We are not aware of empirical work that would allow us to compare the above predictions of our model with empirical findings. Our predictions thus constitute new hypotheses that can be tested in future theoretical and empirical research.

3.4 Existence and uniqueness of equilibrium

The analysis of the expressions (5) and (6) together with Propositions 1 and 3 lead to the existence of a unique linear equilibrium.

[17] This can be seen from the relation $(2a_{i1} - a_{i1}^2)\tau_1 + (2a_{i2} - a_{i2}^2)\tau_2 > (2a_{i2} - a_{i2}^2)\tau_1 + (2a_{i1} - a_{i1}^2)\tau_2$ when $\tau_1 > \tau_2$ and $a_{i1} > a_{i2}$.

Proposition 4. *There exists a unique linear equilibrium, where the equilibrium price is given by* $p = \frac{1}{\Delta + \frac{\tau_\theta}{\Delta \tau_u + \rho}}(\Delta\theta - u)$*, and the suggested strategy by adviser* (i, j) *equals*

$$x_{ij}(y_{ij}, p) = \frac{\mathbb{E}[\theta|y_{ij}, p] - p}{\rho \operatorname{Var}[\theta|y_{ij}, p]} = \rho^{-1}\left(\tau_j y_{ij} - \left(\tau_j + \frac{\rho\tau_\theta}{\Delta\tau_u + \rho}\right) p\right), \tag{12}$$

where Δ *is defined in (4) with the weights* $(a_j)_{j=1,\ldots,n}$ *replaced by the optimal weights* $(a^*_{ij})_{j=1,\ldots,n}$ *in Proposition 3, which are the same across investors, and are independent of the belief* $\widetilde{\tau}^i_\theta$*,* $\widetilde{\tau}^i_u$ *and the aggregation policy* $((\widetilde{a}^i_j)_{j=1,\ldots,n}, \widetilde{\varphi}^i)$ *taken by all other investors.*

3.5 Impact of consultation on market quality

In this subsection, we compare important market quality measures resulting from our model with those from the benchmark economy of Hellwig (1980). We consider the following market quality measures in equilibrium: *Price informativeness* is measured by $1/\operatorname{Var}(\theta|p) = \Delta^2\tau_u$ (Goldstein and Yang 2017; Han and Yang 2013; Ozsoylev and Walden 2011) and reflects the extent to which market prices incorporate information about fundamentals. *Market liquidity* is measured by $\frac{1}{\partial p/\partial(-u)} = \Delta + \frac{\tau_\theta}{\Delta\tau_u + \rho}$. High market liquidity indicates that a shock in supply or noise trading is absorbed without causing significant price fluctuations (Goldstein and Yang 2017; Han and Yang 2013). *Return volatility* is measured by $\sqrt{\operatorname{Var}(\theta - p)}$. We have $\operatorname{Var}(\theta - p) = \left(\frac{\tau_\theta}{(\Delta\tau_u+\rho)^2} + \frac{1}{\tau_u}\right) / \left(\Delta + \frac{\tau_\theta}{\Delta\tau_u+\rho}\right)^2$. These expressions can be derived from the expression $p = \frac{1}{\Delta + \frac{\tau_\theta}{\Delta\tau_u+\rho}}(\Delta\theta - u)$ given in Proposition 4.

The benchmark economy of Hellwig (1980) is identical to the economy in our paper, except that advisers invest directly based on the signals they observe. The equilibrium of the benchmark economy can also be embedded in our economy populated by both investors and their advisers, when investors uniformly aggregate suggested strategies (i.e., $a_{ij} = 1/n$ for any j). This comparison between the two economies is fair, as the precision of the observed signals is identical.

Proposition 5. *Compared with the benchmark economy, price informativeness is (weakly) higher and return volatility is (weakly) lower. Furthermore, market liquidity is (weakly) higher in informationally efficient markets, and investor's expected utility is strictly higher when all advisers have relatively homogeneous signal precision.*[18]

[18] When advisers have different signal precisions, price informativeness and market liquidity are strictly in-

Under the optimal aggregation of suggested strategies, strategies with higher precision receive a higher relative weight than in the benchmark economy. This incorporates more information into prices, thereby improving price informativeness and reducing return volatility. Higher price informativeness means that prices are more indicative of the fundamental value, and as a result, uncertainty about the final payoff is lower. Consequently, investors' strategies become more sensitive to the price. This implies that investors are more willing to provide liquidity, leading to higher market liquidity. Additionally, the optimal aggregation effectively reduces noise in the suggested strategies, resulting in a higher expected utility for a risk-averse investor.

In the benchmark economy, all agents are fully rational and know the true market parameters. In contrast, investors in our model exhibit bounded rationality and do not know these market parameters. A transition from our economy to the benchmark can thus be viewed as a reduction in ambiguity or uncertainty. Conventional wisdom suggests that reducing ambiguity can potentially enhance market participation and generate welfare gains, particularly in contexts where market outcomes are exogenous (Easley and O'Hara 2009, 2010). However, our findings in Proposition 5 indicate that regulatory efforts aimed at reducing ambiguity may have unintended consequences, such as diminished price informativeness, reduced market liquidity in informationally efficient markets, and lower investor welfare. These results highlight the nuanced trade-offs associated with ambiguity reduction in financial markets.

4 Alternative behavioral assumptions

In our main model, investors are subject to three bounded rationality constraints of lack of financial literacy, conformism, and regret aversion under ambiguity. Investors know the signal precision of their advisers but not the true market parameters. In contrast, advisers are fully rational. In this section, we consider alternative constellations of the behavioral assumptions underpinning our analysis. In Subsection 4.1, we consider the case where investors do not know

creasing and return volatility is strictly decreasing. Moreover, when considering a partial equilibrium framework (that is, when the equilibrium price is exogensously given), consultation can always improve investor's expected utility regardless of the difference between advisers' signal precisions.

the signal precision of their advisers and instead adopt a robust approach to formulate optimal aggregation policies. In Subsection 4.2, we examine the case where investors exhibit both a lack of financial literacy and conformism but are not constrained by regret aversion under ambiguity. The reverse relaxation where investors are constrained only by a lack of financial literacy and regret aversion under ambiguity, but not by conformism, is studied in Subsection 4.3. Finally, in Subsection 4.4, we investigate a setting with partially rational advisers who do not infer information about fundamentals from market prices.

The synthesis of this section is that the constraints of bounded rationality imposed on investors in the main body of the paper are essential and cannot be eased while preserving the key finding that investors do not invariably consult with all available advisers. In contrast, all the results in the main body of the paper continue to hold true even when advisers exhibit only partial rationality and disregard information about fundamentals derived from market prices.

4.1 Without knowing signal precisions of advisers

The main model is based on the assumption that investors know the signal precisions of their advisers. In this subsection, we explore how investors optimally aggregate suggested strategies when they do not know individual advisers' precisions. We adapt Definition 1 of admissible aggregation policies to the setting where investors do not know the signal precisions of their advisers as follows.

Definition 3. *Suppose investors do not know the signal precision of advisers. An aggregation policy* $((a_{ij})_{j=1,\dots,n}, \varphi_i) \in \mathbb{R}^n \times \mathbb{R}$ *is called admissible under bounded rationality constraints if*

(i) Lack of financial literacy: the coefficients $(a_{ij})_{j=1,\dots,n}$ *and* φ_i *are constants independent of the signal precision of their advisers* $(\tau_j)_{j=1,\dots,n}$ *and market parameters.*

(ii) Conformism: the investor will buy (or sell) the stock whenever all the advisers suggest buying (or selling) the stock.

(iii) Regret aversion under ambiguity: the aggregated strategy (1) must perform at least as well as the suggested strategy of any single adviser for any belief. Formally, for any $\widetilde{\tau}^i_\theta$, $\widetilde{\tau}^i_u > 0$ *and any aggregation policy* $((\widetilde{a}^i_j)_{j=1,\dots,n}, \widetilde{\varphi}^i)$ *adopted by all other investors, the following*

inequality holds:

$$\begin{aligned}&\inf_{(\tau_j)_{j=1}^n, \frac{1}{n}\sum_{j=1}^n \tau_j = \bar{\tau}_n} \mathbb{E}^{\widetilde{\tau}_\theta^i, \widetilde{\tau}_u^i}_{(\widetilde{a}_1^i, \ldots, \widetilde{a}_n^i, \widetilde{\varphi}^i)} \left[U \left(W \left(\sum_{j=1}^n a_{ij} x_{ij}(y_{ij}(\tau_j), p) + \varphi_i p \right) \right) \right] \\ &\geq \max_{1 \leq j \leq n} \inf_{(\tau_s)_{s=1}^n, \frac{1}{n}\sum_{s=1}^n \tau_s = \bar{\tau}_n} \mathbb{E}^{\widetilde{\tau}_\theta^i, \widetilde{\tau}_u^i}_{(\widetilde{a}_1^i, \ldots, \widetilde{a}_n^i, \widetilde{\varphi}^i)} \left[U \left(W \left(x_{ij}(y_{ij}(\tau_j), p) \right) \right) \right].\end{aligned} \tag{13}$$

The set of aggregation policies admissible under bounded rationality constraints when investors do not know the signal precision of their advisers is denoted by $\bar{\mathcal{A}}$.

There are two key differences compared to Definition 1: First, aggregation policies can no longer depend on the signal precision of advisers. Second, because investors do not know the signal precision of their advisers, they cannot directly compute the expected utility of an aggregation policy even under individual beliefs about market parameters. Instead, investors are ambiguity-averse over all unknown quantities now including the unknown signal precision of advisers and evaluate performance under the worst-case allocation of signal precisions among advisers averaging to a fixed value, $\bar{\tau}_n$. This ambiguity-averse approach is also applied when adapting the definition of equilibrium to the setting in which investors do not know the signal precision of their advisers.

Definition 4. *Suppose investors do not know the signal precision of advisers. An equilibrium is a tuple* $((x_{ij}, a_{ij}^*, \varphi_i^*)_{i=1,\ldots,\infty, j=1,\ldots,n}, p)$ *of strategies suggested by the advisers, aggregation policies in terms of the coefficients in (1), and the price, such that*

(i) Advisers maximize the expected utility of investors: For each i *and* j, x_{ij} *maximizes the expected utility conditional on the private signal* y_{ij} *and price* p, *i.e.,*

$$x_{ij}(y_{ij}, p) \in \arg\max_x \mathbb{E}[U(W(x)) | y_{ij}, p].$$

(ii) Investors optimally aggregate suggested strategies under the constraints of bounded rationality: For each i, *the aggregation policy* $\left((a_{ij}^*)_{j=1,\ldots,n}, \varphi_i^*\right) \in \bar{\mathcal{A}}$ *is admissible under bounded rationality constraints such that for any* $\widetilde{\tau}_\theta^i, \widetilde{\tau}_u^i > 0$ *and* $((\widetilde{a}_j^i)_{j=1,\ldots,n}, \widetilde{\varphi}^i) \in \mathbb{R}^n \times \mathbb{R}$,

it holds that

$$\begin{aligned}&\inf_{(\tau_j)_{j=1}^n, \frac{1}{n}\sum_{j=1}^n \tau_j=\bar{\tau}_n} \mathbb{E}^{\widetilde{\tau}_\theta^i,\widetilde{\tau}_u^i}_{(\widetilde{a}_1^i,\ldots,\widetilde{a}_n^i,\widetilde{\varphi}^i)}\left[U\left(W\left(\sum_{j=1}^n a_{ij}^* x_{ij}(y_{ij}(\tau_j),p)+\varphi_i^* p\right)\right)\right]\\
&\geq \inf_{(\tau_j)_{j=1}^n, \frac{1}{n}\sum_{j=1}^n \tau_j=\bar{\tau}_n} \mathbb{E}^{\widetilde{\tau}_\theta^i,\widetilde{\tau}_u^i}_{(\widetilde{a}_1^i,\ldots,\widetilde{a}_n^i,\widetilde{\varphi}^i)}\left[U\left(W\left(\sum_{j=1}^n a_{ij} x_{ij}(y_{ij}(\tau_j),p)+\varphi_i p\right)\right)\right]\end{aligned}$$

for any $((a_{ij})_{j=1,\ldots,n},\varphi_i)\in\bar{\mathcal{A}}$.

(iii) The market clears:

$$\lim_{h\to\infty}\frac{1}{h}\sum_{i=1}^h\left(\sum_{j=1}^n a_{ij}^* x_{ij}(y_{ij},p)+\varphi_i^* p\right)=u.$$

Admissible strategies under bounded rationality constraints continue to exhibit directional adherence, the sum-of-weights-equals-one heuristic, and price information neglect.

PROPOSITION **6.** *Suppose investors do not know the signal precision of advisers. Then the set of aggregation policies admissible under bounded rationality constraints satisfies*

$$\bar{\mathcal{A}}\subseteq\left\{((a_j)_{j=1,\ldots,n},\varphi)\,\middle|\, a_j\geq 0,\sum_{j=1}^n a_j=1,\varphi=0\right\}.$$

Therefore, the optimal aggregation problem for an individual investor i, who faces uncertainty regarding the quality of advisers, is formulated as follows:

$$\begin{aligned}&\sup_{(a_{ij})_{j=1,\ldots,n}}\ \inf_{(\tau_j)_{j=1,\ldots,n}} \mathbb{E}^{\widetilde{\tau}_\theta^i,\widetilde{\tau}_u^i}_{(\widetilde{a}_1^i,\ldots,\widetilde{a}_n^i,\widetilde{\varphi}^i)}\left[U\left(W\left(\sum_{j=1}^n a_{ij} x_{ij}(y_{ij}(\tau_j),p)\right)\right)\right],\\
&\text{s.t.}\quad \sum_{j=1}^n a_{ij}=1, a_{ij}\geq 0,\\
&\qquad\ \frac{1}{n}\sum_{j=1}^n \tau_j=\bar{\tau}_n.\end{aligned}\tag{14}$$

The following result shows that it is optimal for investors to adopt a simple average of suggested strategies when not knowing the signal precision of advisers.

PROPOSITION **7.** *The optimal solution to optimization problem* (14) *is unique and given by* $a_{ij}^*=1/n$, $j=1,\ldots,n$.

Unlike in the main model, where investors differentiate between suggested strategies by assigning higher weights to those suggested by advisers with higher signal precision, the optimal weight in this setting cannot be contingent on advisers' signal precision. To mitigate the risk of the worst-case consultation, investors aggregate suggested strategies by assigning an equal weight of $1/n$ to each of the n suggested strategies. We remark that the result of Proposition 7 still holds when replacing the constraint on the average signal precision in (14) by

$$\frac{1}{n}\sum_{j=1}^{n}\frac{1}{\tau_j}=K$$

for some $K>0$. Furthermore, similar to Proposition 4, we can establish the existence of a unique equilibrium in the setting where investors do not know the signal precision of their advisers.

We next discuss a more general setting where investors differentiate between advisers. Formally, we consider the constraint

$$\sum_{j=1}^{n} w_{ij}\tau_j = K,\ w_{ij}>0,\ \sum_{j=1}^{n} w_{ij}=1.$$

A larger $w_{ij}>0$ reflects greater relative confidence of investor i in the suggestions of adviser (i,j).

Following a similar procedure as in the symmetric setting, we can define admissible aggregation policies under bounded rationality constraints and market equilibria. Moreover, we can also show that these constraints imply $a_{ij}\geq 0$, $\sum_{j=1}^{n} a_{ij}=1$ and $\varphi_i=0$, leading to a generalized version of (14):

$$\begin{aligned}\sup_{(a_{ij})_{j=1,\dots,n}}\ \inf_{(\tau_j)_{j=1,\dots,n}}\ & \mathbb{E}^{\widetilde{\tau}_\theta^i,\widetilde{\tau}_u^i}_{(\widetilde{a}_1^i,\dots,\widetilde{a}_n^i,\widetilde{\varphi}^i)}\left[U\left(W\left(\sum_{j=1}^{n} a_{ij}x_{ij}(y_{ij}(\tau_j),p)\right)\right)\right]\\ \text{s.t.}\ & \sum_{j=1}^{n} a_{ij}=1, a_{ij}\geq 0,\\ & \sum_{j=1}^{n} w_{ij}\tau_j=K,\end{aligned}\tag{15}$$

where $w_{ij}>0$, $j=1,\dots,n$, with $\sum_{j=1}^{n} w_{ij}=1$. Without loss of generality, we assume that $w_{i1}\geq w_{i2}\geq\cdots\geq w_{in}>0$.

Proposition 8. *The optimal solution to optimization problem (15) is unique and given by the following equalities*

$$\sum_{j=1}^{n}\left(1-\sqrt{1-\frac{w_{ij}}{w_{i1}}(2a_{i1}-a_{i1}^2)}\right)=1,$$

$$a_{ij}=1-\sqrt{1-\frac{w_{ij}}{w_{i1}}(2a_{i1}-a_{i1}^2)},\ j=2,\dots,n.$$

The optimal solution satisfies $a_{i1}^*\geq a_{i2}^*\geq\cdots\geq a_{in}^*>0$.

From Proposition 8, we observe that the optimal aggregation policies maximizing (15) are identical across investors, and indeed independent of $\widetilde{\tau}_\theta^i$, $\widetilde{\tau}_u^i$, and the aggregation policy $((\widetilde{a}_j^i)_{j=1,\dots,n},\widetilde{\varphi}^i)$ taken by all other investors. Moreover, under this non-symmetric setting, we can also show that there exists a unique linear equilibrium. We remark that when investors are unaware of their advisers' signal precision, the optimal aggregation weight assigned to each suggested strategy is strictly positive, regardless of the values of $(w_{ij})_{j=1,\dots,n}$ and $(\tau_j)_{j=1,\dots,n}$. This contrasts with Proposition 3, which shows that when investors do know the signal precision, strategies with low precision may receive an aggregation weight of zero. Intuitively, when investors do not know their advisers' signal precision, they adopt a robust approach to formulating optimal aggregation policies. In this case, every suggested strategy must receive a positive weight. Otherwise, if a particular strategy were assigned a zero weight while its signal fully occupied the available precision capacity and all other strategies had zero precision, the expected utility would be minimized. Moreover, Proposition 8 demonstrates that the optimal aggregation weight a_{ij}^* is higher when the weight w_{ij} assigned to signal precision τ_j in the precision-sum constraint $\sum_{j=1}^n w_{ij}\tau_j=K$ is relatively higher compared to other weights $w_{ir}, r\neq j$. Intuitively, a higher w_{ij} suggests that the feasible signal precision satisfying the precision-sum constraint tends to be lower. To compensate for this, the investor assigns a larger aggregation weight a_{ij} to amplify the term $(2a_{ij}-a_{ij}^2)\tau_j$, thereby maximizing the worst-case expected utility across all feasible signal precisions that satisfy the precision-sum constraint.

4.2 Conformism without regret aversion under ambiguity

In this subsection, we relax the assumptions on bounded rationality and consider investors who exhibit a lack of financial literacy and conformism, but are not constrained by regret aversion under ambiguity.

By Proposition 1, conformism implies directional adherence, meaning $a_{ij} \geq 0$. However, in the absence of regret aversion under ambiguity, investors are not constrained by the sum-of-weights-equals-one heuristic and price information neglect. The optimal aggregation problem (3) is thus equivalent to solving the following optimization problem, as detailed in the proof of Proposition 1 and in particular in (21) in the Appendix:

$$\begin{aligned}\sup_{((a_{ij})_{j=1,\ldots,n},\varphi_i)} & \left(-\frac{\rho\alpha_i}{\beta_i}\left[\varphi_i + \beta_i(1-\bar{a}_i)\right]^2 + 1 + \rho\alpha_i\beta_i + \sum_{j=1}^{n}(2a_{ij} - a_{ij}^2)\tau_j\gamma_i \right) \\ & \text{s.t. } a_{ij} \geq 0, j = 1, \ldots, n,\end{aligned} \tag{16}$$

where $\bar{a}_i = \sum_{j=1}^{n} a_{ij}$, α_i, β_i and γ_i are functions of ρ, $\widetilde{\Delta}^i$, investor i's belief $\widetilde{\tau}_\theta^i$, $\widetilde{\tau}_u^i$, and the aggregation policy $((\widetilde{a}_j^i)_{j=1,\ldots,n}, \widetilde{\varphi}^i)$ adopted by all other investors and independent of the aggregation weight $((a_{ij})_{j=1,\ldots,n}, \varphi_i)$, as shown in the proof of Proposition 1.

Proposition 9. *Suppose investors exhibit only the bounded rationality constraints of lack of financial literacy and conformism, but not regret aversion under ambiguity. Then there exists no equilibrium.*

From the proof of Proposition 9 in the Appendix, we see that the maximum of the optimization problem (16) is achieved when

$$a_{ij} = 1, j = 1, \ldots, n, \quad \varphi_i = (n-1)\beta_i = (n-1)(\widetilde{\tau}_\theta^i + \widetilde{\varphi}^i\widetilde{\Delta}^i\widetilde{\tau}_u^i)/(\widetilde{a}^i\widetilde{\Delta}^i\widetilde{\tau}_u^i + \rho).$$

Investors exhibiting conformism act similarly to fully rational investors, with the important difference that their optimal aggregation policies depend on subjective beliefs $(\widetilde{\tau}_\theta^i, \widetilde{\tau}_u^i)$ instead of the objective market parameters (τ_θ, τ_u). However, the dependence of the optimal solution on investors' subjective beliefs violates the bounded rationality constraint of lacking financial literacy, and as a result, the equilibrium does not exist.

4.3 Regret aversion under ambiguity without conformism

Following the analysis from the previous subsection, we now examine the other relaxation in which investors are constrained only by a lack of financial literacy and regret aversion under ambiguity, but not by conformism. Proposition 1 establishes that $\varphi_i = 0$ and $\sum_{j=1}^n a_{ij} = 1$. Setting $\varphi_i = 0$ and $\bar{a}_i = \sum_{j=1}^n a_{ij} = 1$ in (21) in the proof of Proposition 1 in the Appendix, we see that the optimal aggregation problem (3) is equivalent to solving the following optimization problem:

$$\sup_{(a_{ij})_{j=1,\dots,n}} \left(1 + \rho\alpha_i\beta_i + \sum_{j=1}^n (2a_{ij} - a_{ij}^2)\tau_j\gamma_i\right)$$
$$\text{s.t.} \sum_{j=1}^n a_{ij} = 1,$$

where α_i, β_i and γ_i are given by (19) in the Appendix, depending on the parameters $\widetilde{\tau}_\theta^i$, $\widetilde{\tau}_u^i$, ρ, $\widetilde{\Delta}^i$ and the aggregation policy $((\widetilde{a}_j^i)_{j=1,\dots,n}, \widetilde{\varphi}^i)$ adopted by all other investors, and independent of the aggregation weights $(a_{ij})_{j=1,\dots,n}$. Consequently, the above optimization problem is further equivalent to

$$\begin{aligned} \sup_{(a_{ij})_{j=1,\dots,n}} & \sum_{j=1}^n (2a_{ij} - a_{ij}^2)\tau_j \\ \text{s.t.} & \sum_{j=1}^n a_{ij} = 1. \end{aligned} \tag{17}$$

The optimization problem (17) is identical to objective (11) of the main model, except that here the directional adherence constraint $a_{ij} \geq 0$ is absent.

Proposition 10. *Suppose investors exhibit only the bounded rationality constraints of lack of financial literacy and regret aversion under ambiguity, but not conformism, and $\tau_1 \geq \tau_2 \geq \cdots \geq \tau_n$. Then the optimal aggregation weight, i.e., the optimal solution to the optimization problem (17) is unique and given as follows:*

$$a_{ij}^* = 1 - \frac{n-1}{\tau_j \sum_{r=1}^n \frac{1}{\tau_r}}, \quad j = 1, \dots, n.$$

The solution satisfies $a_{i1}^* \geq a_{i2}^* \geq \cdots \geq a_{in}^*$, where the inequality becomes an equality if and only if the corresponding two signal precisions are identical. Furthermore, $a_{ij}^* < 0$ if and only if $\tau_j < \frac{n-2}{\sum_{r \neq j} \frac{1}{\tau_r}}$. In particular, $a_{in}^* < 0$ if $\tau_n < \frac{n-2}{\sum_{r=1}^{n-1} \frac{1}{\tau_r}}$.

Recall that conformism implies directional adherence, i.e., $a_{ij} \geq 0$. Under this constraint, if the cost of overcounting the impact of the price exceeds the benefit of the additional information content provided by the corresponding suggested strategy, the investor will disregard such a strategy when optimally aggregating the suggested strategies. In the absence of conformism, Proposition 10 shows that the optimal weight assigned to suggested strategies with low relative signal precision can be strictly negative. In particular, an investor exhibiting regret aversion under ambiguity but not conformism reduces his holding in the stock when advisers with low relative signal precision recommend to buy the stock and, vice versa, increases his position in the stock when advisers with low relative signal precision recommend to sell the stock.

To understand this observation, recall that the lack of financial literacy and regret aversion under ambiguity imply the sum-of-weights-equals-one heuristic and price information neglect. The suggestions of advisers with relatively low signal precision contain less additional information from private information, but all suggestions contain an identical component that depends on the price. A suggested strategy with relatively low signal precision can thus be employed to counteract the issue of overcounting the information reflected in the price. Without directional adherence, every suggested strategy serves a purpose: strategies with high relative signal precision are assigned positive weights to enhance their informational content, while strategies with low relative signal precision are assigned negative weights to offset the overcounting effect caused by other strategies with high signal precision.

4.4 Advisers do not learn from prices

Regret aversion under ambiguity leads to an important behavioral implication: investors exhibit price information neglect. They refrain from making additional price-based adjustments when aggregating suggested strategies. Although investors possess the capability to learn from prices, they endogenously choose not to do so, suggesting a limited role of price-based learning within our framework.

In contrast, we assume fully rational advisers who infer fundamental values from both private signals and from market prices. To further explore the role of learning from prices, we now relax this assumption. The resulting model is identical to the main model of the paper, except that advisers now infer fundamental information only through private signals, disregarding price information. This modeling approach follows the Difference of Opinion (DO) paradigm established in the literature (Banerjee 2011; Banerjee et al. 2009; Banerjee and Kremer 2010;

Eyster et al. 2019). Specifically, each adviser (i, j) suggests a strategy $x_{ij}(y_{ij}, p) = \frac{\mathbb{E}[\theta|y_{ij}]-p}{\rho \operatorname{Var}[\theta|y_{ij}]}$ to their client investor i, $j = 1, ..., n$, $i = 1, ..., h$. Similarly, we can define admissible aggregation policies under bounded rationality constraints and equilibrium, as in Definitions 1 and 2, with the only distinction that advisers do not incorporate price information when formulating their suggested strategies.

Through a similar analysis, we find that all the main results from the main model remain valid in this setting. First, similar to (5), we can show that the sequence of equilibrium prices of finite-agent economies converges in probability to $p = \frac{\Delta\theta - u}{\Delta + \rho^{-1} a \tau_\theta - \varphi}$ as $h \to \infty$, where $a = \sum_{j=1}^{n} a_j$, and $((a_j)_{j=1,...,n}, \varphi)$ represents the aggregation policy adopted by all investors in the market. Second, following the reasoning in Proposition 1, we can establish that the bounded rationality constraint of conformism enforces directional adherence (i.e., $a_{ij} \geq 0$), while the lack of financial literacy and regret aversion under ambiguity lead to price information neglect (i.e., $\varphi_i = 0$), and the sum-of-weights-equals-one heuristic (i.e., $\sum_{j=1}^{n} a_{ij} = 1$). Third, similar to Proposition 2, we can derive the expected utility expression for the weighted average of suggested strategies $\mathbb{E}^{\widetilde{\tau}_\theta^i, \widetilde{\tau}_u^i}_{(\widetilde{a}_1^i, ..., \widetilde{a}_n^i, \widetilde{\varphi}^i)}[U(W(\sum_{j=1}^{n} a_{ij} x_{ij}))]$. Based on this, we can reformulate the optimal aggregation problem as the optimization problem in (11). Consequently, Proposition 3 remains valid, and there exists a unique linear equilibrium in this setting. Notably, the optimal aggregation policy of investors with bounded rationality remains identical regardless of whether advisers learn from prices. A detailed outline of the analysis is provided in the Appendix.

In rational expectations equilibrium models, the price plays two roles. First, within the CARA-normality framework, the price serves as a benchmark for investors' decision-making: investors will buy the asset when their evaluation exceeds the price, and sell the asset when their evaluation is below the price. Second, the price acts as a public signal through which investors infer the fundamental value. Since investors also trade based on private signals, the equilibrium price—determined endogenously via the market-clearing condition—incorporates all available information in the market, thereby reflecting the fundamental value. In the modified model, while the first role of the price remains unchanged, the second role disappears. Specifically, although advisers do not extract fundamental information from the price, their suggested strategies are still influenced by the potential price. This preserves a key characteristic of the rational expectations equilibrium framework: asset prices emerge endogenously through the interaction of investors' strategies and the market-clearing condition. The analysis in this subsection demonstrates that the main results of this paper remain robust regardless of whether advisers learn from prices or not. Therefore, the first role of the price is central to our findings, particularly in addressing the issue of overcounting price information, as discussed earlier in Section 3.

5 Conclusions

We consider a classical rational expectations equilibrium economy populated by two types of agents: Investors and their financial advisers. Investors lack full financial literacy and do not know market parameters. Advisers provide strategy recommendations based on their private signal, which investors then aggregate under three constraints of bounded rationality: lack of financial literacy, conformism, and regret aversion under ambiguity.

Our model can explain why investors consult only with a small number of advisers despite the wide array of available sources. The main mechanism is as follows. The behavioral constraints of conformism and regret aversion under ambiguity imply that investors exhibit price information neglect and directional adherence to suggested strategies, and follow the sum-of-weights-equals-one heuristic: They disregard information contained in the price and aggregate suggested strategies by taking a weighted average, with weights positive, summing to one. Intuitively, incorporating information contained in the price would require knowing market parameters, but because of regret aversion under ambiguity, the investor prefers incorporating the correct price dependence already captured in the strategies suggested by advisers. Consequently, investors with bounded rationality optimally disregard some of the suggested strategies and assign higher weights to strategies suggested by their most trusted advisers. This selective aggregation arises because the drawback of overcounting price information outweighs the marginal benefits of including additional signals from advisers with low signal precision.

Our analysis yields two insights with potential policy implications. First, quality of financial advice is more critical than the quantity of advisers consulted. This finding stresses the need to improve professional standards, emphasizing the importance of quality and transparency in regulating financial advisers. Second, while the optimal number of advisers is typically small, it is larger than one, highlighting the importance of fostering a competitive advisory market. Regulatory frameworks should protect individual investors from dependence on a single financial adviser and encourage a diversified and competitive market for financial advice.

Appendix: Proofs

The following lemma is used to compute the expected utility of a quadratic function (see Lemma A.1 in the Appendix in Marín and Rahi (1999)).

LEMMA **2.** *Suppose that z is an n-dimensional normal random vector with mean 0 and positive definite variance-covariance matrix Σ, and B is a symmetric $n \times n$ matrix. Then*

(i) $\mathbb{E}[\exp(-z'Bz)]$ *is well-defined, i.e.,* $\mathbb{E}[\exp(-z'Bz)] < \infty$ *if and only if the matrix* $(\Sigma^{-1} + 2B)$ *is positive definite,*[19] *and*

(ii) $\mathbb{E}[\exp(-z'Bz)] = (\det(I_n + 2\Sigma B))^{-\frac{1}{2}}$ *if the matrix* $(\Sigma^{-1} + 2B)$ *is positive definite,*

where I_n denotes the identity matrix in $\mathbb{R}^n$ and $\det(\cdot)$ *is the determinant operator.*

Proof of Proposition 1

In this proof, we will respectively show that the lack of financial literacy and regret aversion under ambiguity imply price information neglect (i.e., $\varphi_i = 0$) and sum-of-weights-equals-one heuristic (i.e., $\sum_{j=1}^{n} a_{ij} = 1$), and conformism necessitates directional adherence (i.e., $a_{ij} \geq 0$ for any j) in the following three steps.

Step 1. Here we show that the lack of financial literacy and regret aversion under ambiguity imply price information neglect, i.e., $\varphi_i = 0$. We first calculate the expected utility $\mathbb{E}^{\widetilde{\tau}_\theta^i, \widetilde{\tau}_u^i}_{(\widetilde{a}_1^i, \ldots, \widetilde{a}_n^i, \widetilde{\varphi}^i)}[U(W(\sum_{j=1}^{n} a_{ij}x_{ij} + \varphi_i p))]$. Under investor i's belief of $\widetilde{\tau}_\theta^i$, $\widetilde{\tau}_u^i$ and $((\widetilde{a}_j^i)_{j=1,\ldots,n}, \widetilde{\varphi}^i)$, the strategy $\sum_{j=1}^{n} a_{ij}x_{ij} + \varphi_i p$ can be expressed as

$$x_i^* = \sum_{j=1}^{n} a_{ij}x_{ij} + \varphi_i p = \rho^{-1} \left(\bar{\tau}_i \theta + \xi_i - \left(\bar{\tau}_i - \rho\varphi_i + \frac{\bar{a}_i(\widetilde{\tau}_\theta^i + \widetilde{\varphi}^i \widetilde{\Delta}^i \widetilde{\tau}_u^i)}{1 + \rho^{-1}\widetilde{a}^i \widetilde{\Delta}^i \widetilde{\tau}_u^i} \right) p \right),$$

[19] The proof of the "only if" part of Lemma A.1 in Marín and Rahi (1999) is not explicitly detailed, and a formal and rigorous proof is available upon request.

where the second equality follows from (6), p is the price in (5) with the replacement of τ_θ, τ_u, Δ, φ, a with $\widetilde{\tau}_\theta^i$, $\widetilde{\tau}_u^i$, $\widetilde{\Delta}^i$, $\widetilde{\varphi}^i$ and $\widetilde{a}^i$, $\bar{\tau}_i = \sum_{j=1}^n a_{ij}\tau_j$, $\xi_i = \sum_{j=1}^n a_{ij}\tau_j\epsilon_{ij}$, $\bar{a}_i = \sum_{j=1}^n a_{ij}$, $\widetilde{\Delta}^i = \rho^{-1}\sum_{j=1}^n \widetilde{a}_j^i\tau_j$, $\widetilde{a}^i = \sum_{j=1}^n \widetilde{a}_j^i$, and $((\widetilde{a}_j^i)_{j=1,\dots,n}, \widetilde{\varphi}^i)$ is the aggregation policy taken by all other investors in the market.

From the expressions $p = (\widetilde{\Delta}^i\theta - u)/(\widetilde{\Delta}^i + \hat{\beta})$ (see Equation (5)) and $\theta - p = (\hat{\beta}\theta + u)/(\widetilde{\Delta}^i + \hat{\beta})$ with $\hat{\beta} := \frac{\widetilde{a}^i\widetilde{\tau}_\theta^i - \rho\widetilde{\varphi}^i}{\widetilde{a}^i\widetilde{\Delta}^i\widetilde{\tau}_u^i + \rho}$, we see that the variance-covariance matrix Σ of the random vector z and the matrix B satisfying $\rho x_i^*(\theta - p) = z'Bz$ are given by

$$\Sigma = \begin{pmatrix} \gamma_i & 0 & -\alpha_i \\ 0 & \mathrm{Var}(\xi_i) & 0 \\ -\alpha_i & 0 & \frac{(\widetilde{\Delta}^i)^2/\widetilde{\tau}_\theta^i + 1/\widetilde{\tau}_u^i}{(\widetilde{\Delta}^i + \hat{\beta})^2} \end{pmatrix}, \quad B = \begin{pmatrix} \bar{\tau}_i & \frac{1}{2} & -\frac{\rho\bar{\beta}_i}{2} \\ \frac{1}{2} & 0 & 0 \\ -\frac{\rho\bar{\beta}_i}{2} & 0 & 0 \end{pmatrix}, \tag{18}$$

where $\bar{\beta}_i = \bar{a}_i\beta_i - \varphi_i$,

$$\begin{aligned} \alpha_i &= -\,\mathrm{Cov}^{\widetilde{\tau}_\theta^i, \widetilde{\tau}_u^i}_{(\widetilde{a}_1^i,\dots,\widetilde{a}_n^i,\widetilde{\varphi}^i)}(\theta - p, p) = \frac{-\widetilde{\Delta}^i\hat{\beta}/\widetilde{\tau}_\theta^i + 1/\widetilde{\tau}_u^i}{(\widetilde{\Delta}^i + \hat{\beta})^2} = \frac{\rho(\widetilde{a}^i\widetilde{\Delta}^i\widetilde{\tau}_u^i + \rho)(1/\widetilde{\tau}_u^i + \widetilde{\Delta}^i\widetilde{\varphi}^i/\widetilde{\tau}_\theta^i)}{(\widetilde{a}^i(\widetilde{\Delta}^i)^2\widetilde{\tau}_u^i + \widetilde{\Delta}^i\rho - \rho\widetilde{\varphi}^i + \widetilde{a}^i\widetilde{\tau}_\theta^i)^2}, \\ \beta_i &= \frac{\widetilde{\tau}_\theta^i + \widetilde{\varphi}^i\widetilde{\Delta}^i\widetilde{\tau}_u^i}{\widetilde{a}^i\widetilde{\Delta}^i\widetilde{\tau}_u^i + \rho}, \\ \gamma_i &= \mathrm{Var}^{\widetilde{\tau}_\theta^i, \widetilde{\tau}_u^i}_{(\widetilde{a}_1^i,\dots,\widetilde{a}_n^i,\widetilde{\varphi}^i)}(\theta - p) = \frac{\hat{\beta}^2/\widetilde{\tau}_\theta^i + 1/\widetilde{\tau}_u^i}{(\widetilde{\Delta}^i + \hat{\beta})^2} = \frac{(\widetilde{a}^i\widetilde{\tau}_\theta^i - \rho\widetilde{\varphi}^i)^2/\widetilde{\tau}_\theta^i + (\widetilde{a}^i\widetilde{\Delta}^i\widetilde{\tau}_u^i + \rho)^2/\widetilde{\tau}_u^i}{(\widetilde{a}^i(\widetilde{\Delta}^i)^2\widetilde{\tau}_u^i + \widetilde{\Delta}^i\rho - \rho\widetilde{\varphi}^i + \widetilde{a}^i\widetilde{\tau}_\theta^i)^2}. \end{aligned} \tag{19}$$

According to Part (i) of Lemma 2, the inequality (2) holds only if the matrix $\Sigma^{-1} + 2B$ is positive definite for any $\widetilde{\tau}_\theta^i > 0$ and $\widetilde{\tau}_u^i > 0$. As a result, it is necessary that $\det(I_3 + 2B\Sigma) = \det(\Sigma^{-1} + 2B)\det(\Sigma) > 0$ for any $\widetilde{\tau}_\theta^i > 0$ and $\widetilde{\tau}_u^i > 0$. We next show that this determinant condition holds only if $\varphi_i = 0$. We have

$$I_3 + 2B\Sigma = \begin{pmatrix} 1 + 2(\bar{\tau}_i\gamma_i + \frac{\rho\alpha_i\bar{\beta}_i}{2}) & \mathrm{Var}(\xi_i) & 2\phi_i \\ \gamma_i & 1 & -\alpha_i \\ -\rho\bar{\beta}_i\gamma_i & 0 & 1 + \rho\alpha_i\bar{\beta}_i \end{pmatrix}$$

with $\phi_i = -\bar{\tau}_i\alpha_i - \frac{\rho\bar{\beta}_i}{2}\frac{(\widetilde{\Delta}^i)^2/\widetilde{\tau}_\theta^i + 1/\widetilde{\tau}_u^i}{(\widetilde{\Delta}^i + \hat{\beta})^2}$. Expanding the determinant $\det(I_3 + 2B\Sigma)$ along the first row yields

$$\begin{aligned} &\det(I_3 + 2B\Sigma) \\ &= (1 + 2\bar{\tau}_i\gamma_i + \rho\alpha_i\bar{\beta}_i)(1 + \rho\alpha_i\bar{\beta}_i) - \mathrm{Var}(\xi_i)\gamma_i + 2\phi_i\rho\bar{\beta}_i\gamma_i \end{aligned}$$

$$= (1+\rho\alpha_i\bar{\beta}_i)^2 - (\rho\bar{\beta}_i)^2\gamma_i\frac{(\widetilde{\Delta}^i)^2/\widetilde{\tau}_\theta^i + 1/\widetilde{\tau}_u^i}{(\widetilde{\Delta}^i+\hat{\beta})^2} + 2\bar{\tau}_i\gamma_i(1+\rho\alpha_i\bar{\beta}_i) - \mathrm{Var}(\xi_i)\gamma_i - 2\bar{\tau}_i\alpha_i\rho\bar{\beta}_i\gamma_i$$

$$= (1+\rho\alpha_i(\bar{a}_i\beta_i-\varphi_i))^2 - \rho^2(\bar{a}_i\beta_i-\varphi_i)^2\gamma_i\frac{(\widetilde{\Delta}^i)^2/\widetilde{\tau}_\theta^i + 1/\widetilde{\tau}_u^i}{(\widetilde{\Delta}^i+\hat{\beta})^2} + (2\bar{\tau}_i - \mathrm{Var}(\xi_i))\gamma_i$$

$$= \rho^2\left(\alpha_i^2 - \gamma_i\frac{(\widetilde{\Delta}^i)^2/\widetilde{\tau}_\theta^i + 1/\widetilde{\tau}_u^i}{(\widetilde{\Delta}^i+\hat{\beta})^2}\right)\varphi_i^2 - \left(2\rho\alpha_i(1+\rho\alpha_i\bar{a}_i\beta_i) - 2\rho^2\bar{a}_i\beta_i\gamma_i\frac{(\widetilde{\Delta}^i)^2/\widetilde{\tau}_\theta^i + 1/\widetilde{\tau}_u^i}{(\widetilde{\Delta}^i+\hat{\beta})^2}\right)\varphi_i$$

$$+ (1+\rho\alpha_i\bar{a}_i\beta_i)^2 - (\rho\bar{a}_i\beta_i)^2\gamma_i\frac{(\widetilde{\Delta}^i)^2/\widetilde{\tau}_\theta^i + 1/\widetilde{\tau}_u^i}{(\widetilde{\Delta}^i+\hat{\beta})^2} + (2\bar{\tau}_i - \mathrm{Var}(\xi_i))\gamma_i$$

$$= -\frac{\rho^2}{\widetilde{\tau}_\theta^i\widetilde{\tau}_u^i(\widetilde{\Delta}^i+\hat{\beta})^2}\varphi_i^2 - 2\rho\left(\alpha_i - \frac{\rho\bar{a}_i\beta_i}{\widetilde{\tau}_\theta^i\widetilde{\tau}_u^i(\widetilde{\Delta}^i+\hat{\beta})^2}\right)\varphi_i$$

$$+ 1 + 2\rho\alpha_i\beta_i\bar{a}_i - \frac{(\rho\bar{a}_i\beta_i)^2}{\widetilde{\tau}_\theta^i\widetilde{\tau}_u^i(\widetilde{\Delta}^i+\hat{\beta})^2} + (2\bar{\tau}_i - \mathrm{Var}(\xi_i))\gamma_i \tag{20}$$

$$= -\frac{\rho\alpha_i}{\beta_i}\varphi_i^2 - 2\rho\alpha_i(1-\bar{a}_i)\varphi_i + 1 + \rho\alpha_i\beta_i(2\bar{a}_i - (\bar{a}_i)^2) + \sum_{j=1}^n(2a_{ij} - a_{ij}^2)\tau_j\gamma_i, \tag{21}$$

where the second-to-last equality uses the relation

$$\alpha_i^2 - \gamma_i\frac{(\widetilde{\Delta}^i)^2/\widetilde{\tau}_\theta^i + 1/\widetilde{\tau}_u^i}{(\widetilde{\Delta}^i+\hat{\beta})^2} = -\frac{1}{\widetilde{\tau}_\theta^i\widetilde{\tau}_u^i(\widetilde{\Delta}^i+\hat{\beta})^2},$$

and the last equality uses the relation

$$\alpha_i = \frac{\rho\beta_i}{\widetilde{\tau}_\theta^i\widetilde{\tau}_u^i(\widetilde{\Delta}^i+\hat{\beta})^2}. \tag{22}$$

In this proof, we assume without loss of generality that $\beta_i \neq 0$ since we can exclude the values of $\widetilde{\tau}_\theta^i$ and $\widetilde{\tau}_u^i$ for which $\beta_i = 0$ in the following argument.

It follows from (21) that

$$\det(I_3 + 2B\Sigma)/\gamma_i$$

$$= -\frac{\rho\alpha_i}{\beta_i\gamma_i}\varphi_i^2 - \frac{2\rho\alpha_i}{\gamma_i}(1-\bar{a}_i)\varphi_i + \frac{1}{\gamma_i} + \frac{\rho\alpha_i\beta_i}{\gamma_i}(2\bar{a}_i - (\bar{a}_i)^2) + \sum_{j=1}^n(2a_{ij} - a_{ij}^2)\tau_j$$

$$= -\frac{\rho\alpha_i}{\beta_i\gamma_i}\varphi_i^2 - \frac{2\rho\alpha_i}{\gamma_i}(1-\bar{a}_i)\varphi_i - \frac{\rho\alpha_i\beta_i}{\gamma_i}(1-\bar{a}_i)^2 + \frac{1}{\gamma_i} + \frac{\rho\alpha_i\beta_i}{\gamma_i} + \sum_{j=1}^n(2a_{ij} - a_{ij}^2)\tau_j$$

$$= -\frac{\rho\alpha_i}{\beta_i\gamma_i}\left[\varphi_i + \beta_i(1-\bar{a}_i)\right]^2 + \frac{1}{\gamma_i} + \frac{\rho\alpha_i\beta_i}{\gamma_i} + \sum_{j=1}^n(2a_{ij} - a_{ij}^2)\tau_j, \tag{23}$$

where from the definitions of α_i, β_i and γ_i given at the beginning of this proof, we have

$$\frac{\alpha_i}{\beta_i\gamma_i} = \frac{\rho(\widetilde{a}^i\widetilde{\Delta}^i\widetilde{\tau}_u^i + \rho)^2}{(\widetilde{a}^i\widetilde{\tau}_\theta^i - \rho\widetilde{\varphi}^i)^2\widetilde{\tau}_u^i + (\widetilde{a}^i\widetilde{\Delta}^i\widetilde{\tau}_u^i + \rho)^2\widetilde{\tau}_\theta^i}, \tag{24}$$

$$\frac{\alpha_i\beta_i}{\gamma_i} = \frac{\rho(\widetilde{\tau}_\theta^i + \widetilde{\varphi}^i\widetilde{\Delta}^i\widetilde{\tau}_u^i)^2}{(\widetilde{a}^i\widetilde{\tau}_\theta^i - \rho\widetilde{\varphi}^i)^2\widetilde{\tau}_u^i + (\widetilde{a}^i\widetilde{\Delta}^i\widetilde{\tau}_u^i + \rho)^2\widetilde{\tau}_\theta^i}. \tag{25}$$

We see that when $\widetilde{\tau}_\theta^i$ and $\widetilde{\tau}_u^i$ are sufficiently small, β_i is close to zero, $\frac{\alpha_i}{\beta_i\gamma_i}$ is sufficiently large by (24), $\frac{\alpha_i\beta_i}{\gamma_i}$ is bounded for $\widetilde{\tau}_u^i = \mathcal{O}(\sqrt{\widetilde{\tau}_\theta^i})$ by (25). Moreover, with some simple calculations we see that γ_i is sufficiently large regardless of the values of $\widetilde{\varphi}^i$ and $\widetilde{\Delta}^i$. Thus, we can conclude from (23) that $\varphi_i = 0$, since otherwise the term in (21) is negative for appropriately selected $\widetilde{\tau}_\theta^i$ and $\widetilde{\tau}_u^i$.

Step 2. Here we show that the lack of financial literacy and regret aversion under ambiguity imply sum-of-weights-equals-one heuristic, i.e., $\sum_{j=1}^n a_{ij} = 1$. We continue to calculate the expected utility $\mathbb{E}_{(\widetilde{a}_1^i,\ldots,\widetilde{a}_n^i,\widetilde{\varphi}^i)}^{\widetilde{\tau}_\theta^i,\widetilde{\tau}_u^i}[U(W(\sum_{j=1}^n a_{ij}x_{ij} + \varphi_i p))]$. In order to apply Lemma 2, we first assume that the matrix $\Sigma^{-1} + 2B$ is positive definite, and verify it later. Consequently, using Part (ii) of Lemma 2 with $z = (\theta - p, \xi_i, p)'$, and the matrices Σ and B in (18), we obtain

$$\begin{aligned}
&\mathbb{E}_{(\widetilde{a}_1^i,\ldots,\widetilde{a}_n^i,\widetilde{\varphi}^i)}^{\widetilde{\tau}_\theta^i,\widetilde{\tau}_u^i}\left[-\exp\left(-\rho\left(\sum_{j=1}^n a_{ij}x_{ij} + \varphi_i p\right)(\theta - p)\right)\right] \\
&= -(\det(I_3 + 2\Sigma B))^{-\frac{1}{2}} = -(\det(I_3 + 2B\Sigma))^{-\frac{1}{2}},
\end{aligned}$$

where $\det(I_3 + 2B\Sigma)$ is given by (21). With the substitution $a_{ij} = 1$, $a_{ij_0} = 0, j_0 \neq j$, and $\varphi_i = 0$ in (21), we can get the expected utility of the suggested strategy x_{ij}:

$$\mathbb{E}_{(\widetilde{a}_1^i,\ldots,\widetilde{a}_n^i,\widetilde{\varphi}^i)}^{\widetilde{\tau}_\theta^i,\widetilde{\tau}_u^i}\left[-\exp\left(-\rho x_{ij}(\theta - p)\right)\right] = -(1 + \rho\alpha_i\beta_i + \tau_j\gamma_i)^{-\frac{1}{2}}. \tag{26}$$

Therefore, based on (21) with $\varphi_i = 0$ and (26), the inequality (2) is equivalent to

$$1 + \rho\alpha_i\beta_i(2\bar{a}_i - (\bar{a}_i)^2) + \sum_{j=1}^n(2a_{ij} - a_{ij}^2)\tau_j\gamma_i \geq 1 + \rho\alpha_i\beta_i + \max_{1\leq j\leq n}\tau_j\gamma_i,$$

i.e.,

$$-\frac{\rho\alpha_i\beta_i}{\gamma_i}(1 - \bar{a}_i)^2 + \sum_{j=1}^n(2a_{ij} - a_{ij}^2)\tau_j - \max_{1\leq j\leq n}\tau_j \geq 0 \tag{27}$$

for any $\widetilde{\tau}_\theta^i > 0$ and $\widetilde{\tau}_u^i > 0$. Based on (25), when $\widetilde{\tau}_\theta^i$ is sufficiently large and $\widetilde{\tau}_u^i$ is sufficiently small, $\frac{\alpha_i\beta_i}{\gamma_i}$ is sufficiently large. Consequently, (27) holds only when $\bar{a}_i = 1$.

Step 3. Here we show that conformism implies directional adherence, i.e., $a_{ij} \geq 0$ for any j. In fact, this can be directly shown by using a contradiction method.

Finally, we need to show that the matrix $\Sigma^{-1} + 2B$ is positive definite. With some calculations, we have

$$\Sigma^{-1} + 2B = \begin{pmatrix} \frac{(\widetilde{\Delta}^i)^2/\widetilde{\tau}_\theta^i + 1/\widetilde{\tau}_u^i}{\eta(\widetilde{\Delta}^i + \hat{\beta})^2} + 2\bar{\tau}_i & 1 & \frac{\alpha_i}{\eta} - \rho\bar{\beta}_i \\ 1 & \frac{1}{\mathrm{Var}(\xi_i)} & 0 \\ \frac{\alpha_i}{\eta} - \rho\bar{\beta}_i & 0 & \frac{\gamma_i}{\eta} \end{pmatrix},$$

where

$$\begin{aligned} \eta &= \gamma_i \frac{(\widetilde{\Delta}^i)^2/\widetilde{\tau}_\theta^i + 1/\widetilde{\tau}_u^i}{(\widetilde{\Delta}^i + \hat{\beta})^2} - \alpha_i^2 \\ &= \mathrm{Var}^{\widetilde{\tau}_\theta^i, \widetilde{\tau}_u^i}_{(\widetilde{a}_1^i, \ldots, \widetilde{a}_n^i, \widetilde{\varphi}^i)}(\theta - p)\, \mathrm{Var}^{\widetilde{\tau}_\theta^i, \widetilde{\tau}_u^i}_{(\widetilde{a}_1^i, \ldots, \widetilde{a}_n^i, \widetilde{\varphi}^i)}(p) - \left(\mathrm{Cov}^{\widetilde{\tau}_\theta^i, \widetilde{\tau}_u^i}_{(\widetilde{a}_1^i, \ldots, \widetilde{a}_n^i, \widetilde{\varphi}^i)}(\theta - p, p)\right)^2 > 0. \end{aligned}$$

A symmetric matrix is positive definite if and only if all its leading principal minors are positive. It is obvious that the first-order leading principal minor of $\Sigma^{-1} + 2B$ equals $\frac{(\widetilde{\Delta}^i)^2/\widetilde{\tau}_\theta^i + 1/\widetilde{\tau}_u^i}{\eta(\widetilde{\Delta}^i + \hat{\beta})^2} + 2\bar{\tau}_i$, which is positive for any $\widetilde{\tau}_\theta^i > 0$ and $\widetilde{\tau}_u^i > 0$. The second-order leading principal minor of $\Sigma^{-1} + 2B$ satisfies that

$$\left(\frac{(\widetilde{\Delta}^i)^2/\widetilde{\tau}_\theta^i + 1/\widetilde{\tau}_u^i}{\eta(\widetilde{\Delta}^i + \hat{\beta})^2} + 2\bar{\tau}_i\right)\frac{1}{\mathrm{Var}(\xi_i)} - 1 > \frac{2\bar{\tau}_i}{\mathrm{Var}(\xi_i)} - 1 = \frac{2\sum_{j=1}^n a_{ij}\tau_j}{\sum_{j=1}^n a_{ij}^2\tau_j} - 1.$$

As a result, the second-order leading principal minor of $\Sigma^{-1} + 2B$ is positive for any $\widetilde{\tau}_\theta^i > 0$ and $\widetilde{\tau}_u^i > 0$, if $\sum_{j=1}^n (2a_{ij} - a_{ij}^2)\tau_j \geq 0$. Moreover, from (21) we see that the third-order leading principal minor of $\Sigma^{-1} + 2B$ is positive, i.e., $\det(\Sigma^{-1} + 2B) > 0$, or equivalently $\det(I_3 + 2B\Sigma) > 0$ if $\bar{a}_i = \sum_{j=1}^n a_{ij} \leq 2$. These conditions for positive-definiteness are indeed true because we have shown that $a_{ij} \geq 0$ for any j, and $\bar{a}_i = 1$ in the above three steps. The proof is completed. □

Proof of Lemma 1

Follow directly from (5) and the projection theorem for normal variables. □

Proof of Proposition 2

The third expression in (9) is derived from (21) by setting $\varphi_i = 0$ and $\bar{a}_i = 1$. As demonstrated in the proof of Proposition 1, the expected utility at x_{ij} is given by

$$\mathbb{E}^{\widetilde{\tau}_\theta^i, \widetilde{\tau}_u^i}_{(\widetilde{a}_1^i, \ldots, \widetilde{a}_n^i, \widetilde{\varphi}^i)}[-\exp(-\rho x_{ij}(\theta - p))] = -\left(1 + \rho\alpha_i\beta_i + \tau_j\gamma_i\right)^{-\frac{1}{2}}.$$

Consequently, the welfare of each investor adopting the weighted average x_i^* equals that from directly following adviser (i, j)'s suggested strategy if and only if $\tau_j = \sum_{j_0=1}^{n}(2a_{ij_0} - a_{ij_0}^2)\tau_{j_0}$ holds. The second expression in (9) then follows from the alternative expression (7) of the expected utility at x_{ij}. Moreover, the first expression in (9) is obtained by applying the projection theorem for normal random variables to the second expression in (7). The proof is completed. □

Proof of Proposition 3

We begin with Part (i). To economize the notation, here we omit the subscript i in a_{ij}, and instead consider the following constrained optimization problem:

$$\max_{(a_j)_{j=1,\dots,n}} \sum_{j=1}^{n}(2a_j - a_j^2)\tau_j \ \text{ s.t. } \sum_{j=1}^{n} a_j = 1, a_j \geq 0. \tag{28}$$

We establish the following result:

Suppose $\tau_1 \geq \tau_2 \geq \cdots \geq \tau_n > 0$. Then the optimal solution to the optimization problem (28) exists, is unique, has at least two strictly positive components, and is given by

$$a_t^* = 1 - \frac{t-1}{1 + \sum_{\ell=1}^{t-1}\frac{\tau_t}{\tau_\ell}};$$

$$a_j^* = \frac{a_t^*\tau_t + \tau_j - \tau_t}{\tau_j} = 1 - \frac{(t-1)\frac{\tau_t}{\tau_j}}{1 + \sum_{\ell=1}^{t-1}\frac{\tau_t}{\tau_\ell}}, \ j = 1, \dots, t-1;$$

$$a_j^* = 0, \ j = t+1, \dots, n,$$

where $t = \max\{j | 1 \leq j \leq n, 1 + \sum_{\ell=1}^{j-1}\frac{\tau_j - \tau_\ell}{\tau_\ell} > 0\}$. The optimal solution satisfies that $a_1^* \geq a_2^* \geq \cdots \geq a_t^* > 0$ and the inequality becomes equality if and only if the corresponding two signal precisions are identical. In particular, when $\tau_1 = \tau_2 = \cdots = \tau_n$, it holds that $a_1^* = a_2^* = \cdots = a_n^* = 1/n$.

Proof. There exists an optimal solution $(a_j^*)_{j=1,\dots,n}$ with $0 \leq a_j^* \leq 1$ to this constrained optimization problem since the constraint set is a bounded, closed set and the objective function is continuous. Moreover, the optimal solution is unique since the objective function is strictly convex.

We now derive the necessary conditions that the optimal solution satisfies. We claim that for any i and j with $a_j^* > 0$, it must hold that $a_i^*\tau_i - a_j^*\tau_j = \tau_i - \tau_j$. Let $0 < \delta < a_j^*$ and consider the feasible solution where the i-th component is $a_i^* + \delta$, the j-th component is $a_j^* - \delta$ and the other components equal $a_\ell^*, \ell \neq i, j$. The objective value for this feasible solution is

$$2(a_i^* + \delta)\tau_i - (a_i^* + \delta)^2\tau_i + 2(a_j^* - \delta)\tau_j - (a_j^* - \delta)^2\tau_j + \sum_{\ell \neq i,j}(2a_\ell^* - (a_\ell^*)^2)\tau_\ell,$$

which achieves its maximum at $\delta = 0$. Taking derivative at $\delta = 0$ leads to the claim. The claim implies that (i) any solution with a single component equal to one and all other components equal to zero cannot be optimal. In other words, the optimal solution must contain at least two positive components. (ii) $a_i^*\tau_i \geq a_j^*\tau_j > 0$ whenever $\tau_i \geq \tau_j$ and $a_j^* > 0$, and (iii) if $a_j^* > 0$, then $a_i^* \geq a_j^* > 0$ for all $i \leq j$ (otherwise, if $a_i^* < a_j^*$, then $a_i^*\tau_i - a_j^*\tau_j < a_j^*(\tau_i - \tau_j) \leq \tau_i - \tau_j$, leading to a contradiction), and $a_i^* = a_j^*$ if and only if $\tau_i = \tau_j$. That is, when the optimal weight assigned to a low-precision signal is positive, the corresponding weight assigned to a high-precision signal must be strictly larger.

We claim that $a_j^* = 0$ for all $j \geq t+1$. Otherwise, let $s = \max\{j | t+1 \leq j \leq n, a_j^* > 0\}$, then from the relation $a_j^*\tau_j - a_s^*\tau_s = \tau_j - \tau_s$ for $j \leq s$, we have $a_j^* = \frac{a_s^*\tau_s + \tau_j - \tau_s}{\tau_j}, j = 1, \ldots, s$. By the definition of s, $a_j^* = 0$ for $j \geq s+1$. Thus,

$$\sum_{\ell=1}^{n} a_\ell^* = \sum_{\ell=1}^{s} a_\ell^* = \sum_{\ell=1}^{s} \frac{a_s^*\tau_s + \tau_\ell - \tau_s}{\tau_\ell} = 1.$$

We can solve $a_s^* = \frac{1 + \sum_{\ell=1}^{s-1} \frac{\tau_s - \tau_\ell}{\tau_\ell}}{1 + \sum_{\ell=1}^{s-1} \frac{\tau_s}{\tau_\ell}}$, which is nonpositive from the definition of t, but positive by the definition of s, a contradiction. Thus, $a_j^* = 0$ for all $j \geq t+1$. Similar to the above arguments, we can solve

$$a_t^* = \frac{1 + \sum_{\ell=1}^{t-1} \frac{\tau_t - \tau_\ell}{\tau_\ell}}{1 + \sum_{\ell=1}^{t-1} \frac{\tau_t}{\tau_\ell}} = 1 - \frac{t-1}{1 + \sum_{\ell=1}^{t-1} \frac{\tau_t}{\tau_\ell}},$$

which is positive by the definition of t, and

$$a_j^* = \frac{a_t^*\tau_t + \tau_j - \tau_t}{\tau_j} = 1 - \frac{(t-1)\frac{\tau_t}{\tau_j}}{1 + \sum_{\ell=1}^{t-1} \frac{\tau_t}{\tau_\ell}}$$

for $j = 1, \ldots, t-1$ as given in the lemma based on the established relation $a_j^*\tau_j - a_t^*\tau_t = \tau_j - \tau_t$. Moreover, the relation $a_1^* \geq a_2^* \geq \cdots \geq a_t^*$ follows from the result that if $a_j^* > 0$, then $a_i^* \geq a_j^* > 0$ for all $i \leq j$ we have shown in the first paragraph. The last part is straightforward.

Taking the notations in Part (i), we now show Part (ii). Differentiating with respect to τ_1 yields

$$\frac{\partial \sum_{j=1}^{n}(2a_j^* - (a_j^*)^2)\tau_j}{\partial \tau_1} = 2a_1^* - (a_1^*)^2 + (2 - 2a_1^*)\frac{\partial a_1^*}{\partial \tau_1}\tau_1 + \sum_{j \neq 1}(2 - 2a_j^*)\frac{\partial a_j^*}{\partial \tau_1}\tau_j. \tag{29}$$

By Part (i) of Proposition 3, we have

$$\frac{\partial a_t^*}{\partial \tau_1} = -\frac{t-1}{(1+\sum_{\ell=1}^{t-1}\frac{\tau_t}{\tau_\ell})^2}\frac{\tau_t}{\tau_1^2},$$
$$\frac{\partial a_j^*}{\partial \tau_1} = \frac{\partial a_t^*}{\partial \tau_1}\frac{\tau_t}{\tau_j}, j \neq 1,$$
$$\frac{\partial a_1^*}{\partial \tau_1} = \frac{\partial[(a_t^*-1)\frac{\tau_t}{\tau_1}]}{\partial \tau_1} = \frac{\partial a_t^*}{\partial \tau_1}\frac{\tau_t}{\tau_1} - (a_t^*-1)\frac{\tau_t}{\tau_1^2}.$$

As a result,

$$\begin{aligned}(2-2a_1^*)\frac{\partial a_1^*}{\partial \tau_1}\tau_1 + \sum_{j\neq 1}(2-2a_j^*)\frac{\partial a_j^*}{\partial \tau_1}\tau_j &= \sum_{j=1}^{t}(2-2a_j^*)\frac{\partial a_t^*}{\partial \tau_1}\tau_t + (2-2a_1^*)(1-a_t^*)\frac{\tau_t}{\tau_1} \\ &= -\frac{(2t-2)(t-1)}{(1+\sum_{\ell=1}^{t-1}\frac{\tau_t}{\tau_\ell})^2}\frac{\tau_t^2}{\tau_1^2} + 2\frac{(t-1)\frac{\tau_t}{\tau_1}}{1+\sum_{\ell=1}^{t-1}\frac{\tau_t}{\tau_\ell}}\frac{t-1}{1+\sum_{\ell=1}^{t-1}\frac{\tau_t}{\tau_\ell}}\frac{\tau_t}{\tau_1} \\ &= 0. \end{aligned} \tag{30}$$

From (29) and (30), we obtain

$$\frac{\partial \sum_{j=1}^{n}(2a_j^* - (a_j^*)^2)\tau_j}{\partial \tau_1} = 2a_1^* - (a_1^*)^2 > 0.$$

The sensitivity analysis with respect to other τ_j is similar and omitted. The proof is completed. □

Proof of Proposition 4

It follows from Proposition 1 that $\varphi = 0$. The existence and uniqueness of equilibrium follow directly from substituting φ, $(a_j)_{j=1,\dots,n}$ in the expressions (4), (5) and (6) with zero and the optimal weight solution given in Proposition 3, respectively. Notably, the optimal aggregation policy is independent of $\widetilde{\tau}_\theta^i$, $\widetilde{\tau}_u^i$ and the aggregation policy $((\widetilde{a}_j^i)_{j=1,\dots,n}, \widetilde{\varphi}^i)$ adopted by all the other investors, and the same across investors. The proof is completed. □

Proof of Proposition 5

We first show that consultation increases Δ and thereby improves price informativeness. From the expression of a_{ij}^* in Proposition 3 and (4), it suffices to show that

$$\sum_{j=1}^{n} a_{ij}^*\tau_j = \sum_{j=1}^{t}\left(1 - \frac{(t-1)\frac{\tau_t}{\tau_j}}{1+\sum_{\ell=1}^{t-1}\frac{\tau_t}{\tau_\ell}}\right)\tau_j \geq \sum_{j=1}^{n}\tau_j/n,$$

which is equivalent to

$$(n-1)(\tau_1+\tau_2+\cdots+\tau_t) \geq \frac{(t-1)\tau_t}{\sum_{\ell=1}^{t}\frac{\tau_t}{\tau_\ell}}tn+\tau_{t+1}+\tau_{t+2}+\cdots+\tau_n.$$

This inequality holds due to the following relations

$$(\tau_1+\tau_2+\cdots+\tau_t)\sum_{\ell=1}^{t}\frac{\tau_t}{\tau_\ell} \geq t^2\tau_t, \quad \tau_{t+1}+\tau_{t+2}+\cdots+\tau_n \leq (n-t)\tau_t,$$

and $\sum_{\ell=1}^{t}\frac{\tau_t}{\tau_\ell} \leq t$. The claim follows.

Second, recall that $\mathrm{Var}(\theta-p)=\left(\frac{\hat{\beta}^2}{\tau_\theta}+\frac{1}{\tau_u}\right)/(\Delta+\hat{\beta})^2$, where $\hat{\beta}=\frac{\tau_\theta}{\Delta\tau_u+\rho}$. Direct computation yields

$$\begin{aligned}
\frac{\partial\,\mathrm{Var}(\theta-p)}{\partial\Delta} &= \frac{2\frac{\hat{\beta}}{\tau_\theta}\frac{\partial\hat{\beta}}{\partial\Delta}(\Delta+\hat{\beta})^2-2(\frac{\hat{\beta}^2}{\tau_\theta}+\frac{1}{\tau_u})(\Delta+\hat{\beta})(1+\frac{\partial\hat{\beta}}{\partial\Delta})}{(\Delta+\hat{\beta})^4}\\
&= \frac{2}{(\Delta+\hat{\beta})^3}\left(\frac{\hat{\beta}}{\tau_\theta}\frac{\partial\hat{\beta}}{\partial\Delta}(\Delta+\hat{\beta})-\left(\frac{\hat{\beta}^2}{\tau_\theta}+\frac{1}{\tau_u}\right)\left(1+\frac{\partial\hat{\beta}}{\partial\Delta}\right)\right)\\
&= \frac{2}{(\Delta+\hat{\beta})^3}\left(\frac{\hat{\beta}}{\tau_\theta}\frac{\partial\hat{\beta}}{\partial\Delta}\Delta-\frac{\hat{\beta}^2}{\tau_\theta}-\frac{1}{\tau_u}-\frac{1}{\tau_u}\frac{\partial\hat{\beta}}{\partial\Delta}\right)\\
&= \frac{2}{(\Delta+\hat{\beta})^3}\left(\left(\frac{\Delta}{\Delta\tau_u+\rho}-\frac{1}{\tau_u}\right)\frac{\partial\hat{\beta}}{\partial\Delta}-\frac{1}{\tau_u}-\frac{\tau_\theta}{(\Delta\tau_u+\rho)^2}\right)\\
&= -\frac{2}{(\Delta+\hat{\beta})^3}\left(\frac{\Delta}{\Delta\tau_u+\rho}\frac{\tau_\theta\tau_u}{(\Delta\tau_u+\rho)^2}+\frac{1}{\tau_u}\right)\\
&< 0,
\end{aligned}$$

where we use the relation $\frac{\partial\hat{\beta}}{\partial\Delta}=-\frac{\tau_\theta\tau_u}{(\Delta\tau_u+\rho)^2}$. Since consultation increases Δ, we conclude that it decreases return volatility.

Third, direct calculation shows that

$$\frac{\partial\left(\Delta+\frac{\tau_\theta}{\Delta\tau_u+\rho}\right)}{\partial\Delta}=1-\frac{\tau_\theta\tau_u}{(\Delta\tau_u+\rho)^2},$$

which is positive when Δ is sufficiently large. Thus, consultation improves market liquidity in informationally efficient markets.

Finally, to establish the last conclusion, it suffices to consider the case in which all advisers in the economy possess the same signal precision. From (4) and the definition of the benchmark economy, we see that consultation does not affect Δ, and hence does not affect p. Consequently, equilibrium welfare improves due to (9), (10) and the following relation

$$\sum_{j=1}^{n}(2a_{ij}^{*}-(a_{ij}^{*})^{2})\tau_j \geq \sum_{j=1}^{n}(2/n-1/n^2)\tau_j > \sum_{j=1}^{n}\tau_j/n.$$

The proof is completed. $\square$

Proof of Proposition 6

Following the notations and a similar line of reasoning as in the proof of Proposition 1, we obtain from equation (13) the following inequality:

$$\begin{aligned}&-\frac{\rho\alpha_i}{\beta_i\gamma_i}\varphi_i^2-\frac{2\rho\alpha_i}{\gamma_i}(1-\bar{a}_i)\varphi_i-\frac{\rho\alpha_i\beta_i}{\gamma_i}(1-\bar{a}_i)^2\\&\quad+\min_{(\tau_j)_{j=1,\dots,n},\frac{1}{n}\sum_{j=1}^{n}\tau_j=\bar{\tau}_n}\sum_{j=1}^{n}(2a_{ij}-a_{ij}^2)\tau_j-\max_{1\leq j\leq n}\min_{(\tau_s)_{s=1,\dots,n},\frac{1}{n}\sum_{s=1}^{n}\tau_s=\bar{\tau}_n}\tau_j>0.\end{aligned}$$

Based on this inequality, we can apply similar arguments as in the proof of Proposition 1 to conclude that $\bar{a}_i=\sum_{j=1}^{n}a_{ij}=1$ and $\varphi_i=0$. $\square$

Proof of Proposition 7

By virtue of Proposition 2, the optimization problem (14) is equivalent to

$$\begin{aligned}\sup_{(a_{ij})_{j=1,\dots,n}}\ &\inf_{(\tau_j)_{j=1,\dots,n}}\ \sum_{j=1}^{n}(2a_{ij}-a_{ij}^2)\tau_j\\ \text{s.t.}\ &\sum_{j=1}^{n}a_{ij}=1, a_{ij}\geq 0,\\ &\frac{1}{n}\sum_{j=1}^{n}\tau_j=\bar{\tau}_n\end{aligned}\tag{31}$$

Fix $\{a_{ij}\}_{j=1,\dots,n}$ and consider the inner optimization problem $\min_{\tau_j,j=1,\dots,n}\sum_{j=1}^{n}(2a_{ij}-a_{ij}^2)\tau_j$. The lowest value of $(2a_{ij}-a_{ij}^2)$ over j will receive all the total precision $n\bar{\tau}_n$, while the others are assigned zero precision. That is, $\tau_{j_1}=n\bar{\tau}_n$ for $j_1\in\arg\min_{1\leq j\leq n}(2a_{ij}-a_{ij}^2)$, and $\tau_j=0$ for $j\neq j_1$. Therefore, the robust optimization problem (31) reduces to $\max_{a_{ij},j=1,\dots,n}\min_{1\leq j\leq n}\ (2a_{ij}-a_{ij}^2)$, which clearly attains its optimum at $a_{ij}=1/n$ for all j. The proof is completed. $\square$

Proof of Proposition 8

With the change of variable $\tau_j' = w_{ij}\tau_j$, the robust optimization problem (15) can be transferred into:

$$\begin{aligned} \sup_{(a_{ij})_{j=1,\dots,n}} \inf_{(\tau_j')_{j=1,\dots,n}} \quad & \sum_{j=1}^{n} \frac{2a_{ij} - a_{ij}^2}{w_{ij}} \tau_j' \\ \text{s.t.} \quad & \sum_{j=1}^{n} a_{ij} = 1, a_{ij} \geq 0, \\ & \sum_{j=1}^{n} \tau_j' = K. \end{aligned} \tag{32}$$

Fix $\{a_{ij}\}_{j=1,\dots,n}$ and consider the following optimization problem:

$$\min_{(\tau_j')_{j=1,\dots,n}} \sum_{j=1}^{n} \frac{2a_{ij} - a_{ij}^2}{w_{ij}} \tau_j'.$$

The component with the lowest value of $(2a_{ij} - a_{ij}^2)/w_{ij}$ over j will receive the entire precision K, while all others will be assigned zero precision. That is, $\tau_{j_1}' = K$ for $j_1 = \arg\min_{1\leq j\leq n}(2a_{ij} - a_{ij}^2)/w_{ij}$, and $\tau_j' = 0$ for $j \neq j_1$. Then, the robust optimization problem (32) reduces to $\sup_{(a_{ij})_{j=1,\dots,n}} \inf_{1\leq j\leq n} \ (2a_{ij} - a_{ij}^2)/w_{ij}$, which we next solve.

We claim that the optimal solution (still denoted by a_{ij}'s) must satisfy $a_{ij} > 0$ for any j, and that

$$\frac{2a_{i1} - a_{i1}^2}{w_{i1}} = \frac{2a_{i2} - a_{i2}^2}{w_{i2}} = \cdots = \frac{2a_{in} - a_{in}^2}{w_{in}}. \tag{33}$$

Otherwise, we can increase a_{ij_1} by a small amount δ for some $j_1 \in \arg\min_{1\leq j\leq n} \frac{2a_{ij} - a_{ij}^2}{w_{ij}}$, and simultaneously decrease a_{ij_2} by δ for some $\frac{2a_{ij_2} - a_{ij_2}^2}{w_{ij_2}} > \frac{2a_{ij_1} - a_{ij_1}^2}{w_{ij_1}}$ in order to increase the minimum value of $\left\{\frac{2a_{ij} - a_{ij}^2}{w_{ij}}\right\}_{j=1,\dots,n}$.

Note that $w_{i1} \geq w_{i2} \geq \cdots \geq w_{in}$. It then follows from (33) that $a_{i1} \geq a_{i2} \geq \cdots \geq a_{in}$. From (33), we further obtain

$$a_{ij} = 1 - \sqrt{1 - \frac{w_{ij}}{w_{i1}}(2a_{i1} - a_{i1}^2)}, j = 2, \dots, n. \tag{34}$$

This is intuitive, as a larger $w_{ij} > 0$ reflects greater relative confidence of investor i in the suggestions of adviser (i, j).

Finally, we solve for a_{i1} using the constraint $\sum_{j=1}^n a_{ij} = 1$, that is,

$$\sum_{j=1}^{n}\left(1-\sqrt{1-\frac{w_{ij}}{w_{i1}}(2a_{i1}-a_{i1}^2)}\right)=1,$$

from which we can first determine a unique solution $0 < a_{i1} < 1$, and then a_{ij} using (34). The proof is completed. □

Proof of Proposition 9

From the proof of Proposition 1, we see that α_i, β_i and γ_i are functions of ρ, $\widetilde{\Delta}^i$, investor i's belief parameters $\widetilde{\tau}_\theta^i$, $\widetilde{\tau}_u^i$, as well as the aggregation policy $((\widetilde{a}_j^i)_{j=1,\ldots,n}, \widetilde{\varphi}^i)$ adopted by all other investors. Importantly, they are independent of investor i's own aggregation weights $((a_{ij})_{j=1,\ldots,n}, \varphi_i)$. Moreover, it follows from relations (22) and (19) that both α_i/β_i and γ_i are positive. Also note that $2a_{ij} - a_{ij}^2 \leq 1$ for any a_{ij}. Consequently,

$$-\frac{\rho\alpha_i}{\beta_i}\left[\varphi_i+\beta_i(1-\bar{a}_i)\right]^2+1+\rho\alpha_i\beta_i+\sum_{j=1}^{n}(2a_{ij}-a_{ij}^2)\tau_j\gamma_i\leq 1+\rho\alpha_i\beta_i+\sum_{j=1}^{n}\tau_j\gamma_i$$

for any a_{ij} and φ_i, where the upper bound is achieved when $a_{ij} = 1, j = 1, \ldots, n$ and $\varphi_i = (n-1)\beta_i = (n-1)(\widetilde{\tau}_\theta^i + \widetilde{\varphi}^i\widetilde{\Delta}^i\widetilde{\tau}_u^i)/(\widetilde{a}^i\widetilde{\Delta}^i\widetilde{\tau}_u^i + \rho)$. That is, the policy $a_{ij}^* = 1, j = 1, \ldots, n,\ \ \varphi_i^* = (n-1)\beta_i = (n-1)(\widetilde{\tau}_\theta^i + \widetilde{\varphi}^i\widetilde{\Delta}^i\widetilde{\tau}_u^i)/(\widetilde{a}^i\widetilde{\Delta}^i\widetilde{\tau}_u^i + \rho)$ is the optimal solution to the optimization problem (16). However, such an optimal solution violates the bounded rationality constraint of lack of financial literacy, which requires that an investor's aggregation policy cannot depend on their own beliefs. Therefore, an equilibrium does not exist. This completes the proof. □

Proof of Proposition 10

The Lagrangian for the optimization problem (11) is given by

$$\mathcal{L}(a_{i1},...,a_{in},\lambda) := \sum_{j=1}^{n}(2a_{ij}-a_{ij}^2)\tau_j - \lambda\left(\sum_{j=1}^{n}a_{ij}-1\right).$$

Setting $\frac{\partial\mathcal{L}}{\partial a_{ij}} = 0$ yields $2(1-a_{ij})\tau_j = \lambda, j = 1, ..., n$. This implies $a_{ij} = 1 - \frac{\lambda}{2\tau_j},\ j = 1, ..., n$. From the constraint $\sum_{j=1}^n a_{ij} = 1$, we obtain

$$\sum_{j=1}^{n}\left(1-\frac{\lambda}{2\tau_j}\right)=1.$$

Solving for the Lagrange multiplier gives $\lambda = \frac{n-1}{\sum_{j=1}^{n}\frac{1}{2\tau_j}}$. Substituting back, the optimal weights are

$$a_{ij}^{*} = 1 - \frac{\lambda}{2\tau_j} = 1 - \frac{n-1}{\tau_j \sum_{r=1}^{n}\frac{1}{\tau_r}}, j = 1, ..., n.$$

The proof is completed. □

Proof of results in Subsection 4.4

We first derive the limit equilibrium of the large economy. When advisers do not learn from the price, the strategy suggested by adviser (i, j) is

$$x_{ij}(y_{ij}, p) = \frac{\mathbb{E}[\theta|y_{ij}] - p}{\rho\,\mathrm{Var}[\theta|y_{ij}]} = \rho^{-1}\left(\tau_j y_{ij} - (\tau_\theta + \tau_j)\,p\right). \tag{35}$$

Then the market-clearing condition becomes

$$\frac{1}{h}\sum_{i=1}^{h}\left(\sum_{j=1}^{n} a_j x_{ij} + \varphi p\right) = \frac{1}{h}\sum_{i=1}^{h}\left(\sum_{j=1}^{n} a_j[\rho^{-1}\left(\tau_j y_{ij} - (\tau_\theta + \tau_j)\,p\right)] + \varphi p\right) = u.$$

Taking $h \to \infty$ leads to

$$\left(\sum_{j=1}^{n} a_j\left(\rho^{-1}\left(\tau_j\theta - (\tau_\theta + \tau_j)\,p\right)\right) + \varphi p\right) = u.$$

Solving for price gives

$$p = \frac{\Delta\theta - u}{\rho^{-1}\sum_{j=1}^{n} a_j(\tau_\theta + \tau_j) - \varphi} = \frac{\Delta\theta - u}{\Delta + \rho^{-1} a\tau_\theta - \varphi}, \tag{36}$$

where $\Delta = \rho^{-1}\sum_{j=1}^{n} a_j\tau_j$, and $a = \sum_{j=1}^{n} a_j$.

We now show that the bounded rationality constraint of conformism enforces directional adherence (i.e., $a_{ij} \geq 0$), and the lack of financial literacy and regret aversion under ambiguity lead to price information neglect (i.e., $\varphi_i = 0$), and sum-of-weights-equals-one heuristic (i.e., $\sum_{j=1}^{n} a_{ij} = 1$). The following proof adopts similar notation and follows an analogous procedure to that used in the proof of Proposition 1. We first calculate the expected utility $\mathbb{E}^{\widetilde{\tau}_\theta^i, \widetilde{\tau}_u^i}_{(\widetilde{a}_1^i, \dots, \widetilde{a}_n^i, \widetilde{\varphi}^i)}[U(W(\sum_{j=1}^{n} a_{ij}x_{ij} + \varphi_i p))]$, where $((\widetilde{a}_j^i)_{j=1,\dots,n}, \widetilde{\varphi}^i)$ is the aggregation policy taken by all other investors in the market. Under investor i's belief parameters of $\widetilde{\tau}_\theta^i$, $\widetilde{\tau}_u^i$ and $((\widetilde{a}_j^i)_{j=1,\dots,n}, \widetilde{\varphi}^i)$, the aggregated strategy $\sum_{j=1}^{n} a_{ij}x_{ij} + \varphi_i p$ can be expressed as

$$x_i^* := \sum_{j=1}^{n} a_{ij}x_{ij} + \varphi_i p$$

$$= \sum_{j=1}^{n} a_{ij}[\rho^{-1}\left(\tau_j y_{ij} - \left(\widetilde{\tau}_\theta^i + \tau_j\right) p\right)] + \varphi_i p$$
$$= \rho^{-1}\left(\bar{\tau}_i\theta + \xi_i - \left(\bar{\tau}_i - \rho\varphi_i + \bar{a}_i\widetilde{\tau}_\theta^i\right) p\right),$$

where the second equality follows from (35) with the replacement of τ_θ with $\widetilde{\tau}_\theta^i$, $\bar{\tau}_i = \sum_{j=1}^n a_{ij}\tau_j$, $\xi_i = \sum_{j=1}^n a_{ij}\tau_j\epsilon_{ij}$ and $\bar{a}_i = \sum_{j=1}^n a_{ij}$.

We intend to apply Lemma 2 for $z = (\theta - p, \xi_i, p)'$. From the expressions $p = (\widetilde{\Delta}^i\theta - u)/(\widetilde{\Delta}^i + \hat{\beta})$ (see Equation (36) with the replacement of τ_θ, τ_u, Δ, φ, a with $\widetilde{\tau}_\theta^i$, $\widetilde{\tau}_u^i$, $\widetilde{\Delta}^i$, $\widetilde{\varphi}^i$ and $\widetilde{a}^i$) and $\theta - p = (\hat{\beta}\theta + u)/(\widetilde{\Delta}^i + \hat{\beta})$ with $\hat{\beta} = \rho^{-1}\widetilde{a}^i\widetilde{\tau}_\theta^i - \widetilde{\varphi}^i$, $\widetilde{\Delta}^i = \rho^{-1}\sum_{j=1}^n \widetilde{a}_j^i\tau_j$ and $\widetilde{a}^i = \sum_{j=1}^n \widetilde{a}_j^i$, we see that the variance-covariance matrix Σ of the random vector z and the matrix B satisfying $\rho x_i^*(\theta - p) = z'Bz$ are given by

$$\Sigma = \begin{pmatrix} \gamma_i & 0 & -\alpha_i \\ 0 & \mathrm{Var}(\xi_i) & 0 \\ -\alpha_i & 0 & \frac{(\widetilde{\Delta}^i)^2/\widetilde{\tau}_\theta^i + 1/\widetilde{\tau}_u^i}{(\widetilde{\Delta}^i+\hat{\beta})^2} \end{pmatrix}, \quad B = \begin{pmatrix} \bar{\tau}_i & \frac{1}{2} & \frac{\rho\varphi_i - \bar{a}_i\widetilde{\tau}_\theta^i}{2} \\ \frac{1}{2} & 0 & 0 \\ \frac{\rho\varphi_i - \bar{a}_i\widetilde{\tau}_\theta^i}{2} & 0 & 0 \end{pmatrix},$$

where $\alpha_i = \frac{-\widetilde{\Delta}^i\hat{\beta}/\widetilde{\tau}_\theta^i + 1/\widetilde{\tau}_u^i}{(\widetilde{\Delta}^i+\hat{\beta})^2}$ and $\gamma_i = \frac{\hat{\beta}^2/\widetilde{\tau}_\theta^i + 1/\widetilde{\tau}_u^i}{(\widetilde{\Delta}^i+\hat{\beta})^2}$.

With some calculations, we have

$$I_3 + 2B\Sigma = \begin{pmatrix} 1 + 2(\bar{\tau}_i\gamma_i + \frac{\alpha_i(\bar{a}_i\widetilde{\tau}_\theta^i - \rho\varphi_i)}{2}) & \mathrm{Var}(\xi_i) & 2\phi_i \\ \gamma_i & 1 & -\alpha_i \\ (\rho\varphi_i - \bar{a}_i\widetilde{\tau}_\theta^i)\gamma_i & 0 & 1 + \alpha_i(\bar{a}_i\widetilde{\tau}_\theta^i - \rho\varphi_i) \end{pmatrix},$$

where $\phi_i = -\bar{\tau}_i\alpha_i - \frac{\rho\bar{\beta}_i}{2}\frac{(\widetilde{\Delta}^i)^2/\widetilde{\tau}_\theta^i + 1/\widetilde{\tau}_u^i}{(\widetilde{\Delta}^i+\hat{\beta})^2} = -\bar{\tau}_i\alpha_i + \frac{\rho\varphi_i - \bar{a}_i\widetilde{\tau}_\theta^i}{2}\frac{(\widetilde{\Delta}^i)^2/\widetilde{\tau}_\theta^i + 1/\widetilde{\tau}_u^i}{(\widetilde{\Delta}^i+\hat{\beta})^2}$ and $\bar{\beta}_i = \rho^{-1}\bar{a}_i\widetilde{\tau}_\theta^i - \varphi_i$. Denote $\beta_i = \rho^{-1}\widetilde{\tau}_\theta^i$. Expanding the determinant $\det(I_3 + 2B\Sigma)$ along the first row, we see that Equation (20) holds here:

$$\begin{aligned} \det(I_3 + 2B\Sigma) &= -\frac{\rho^2}{\widetilde{\tau}_\theta^i\widetilde{\tau}_u^i(\widetilde{\Delta}^i + \hat{\beta})^2}\varphi_i^2 - 2\rho\left(\alpha_i - \frac{\rho\bar{a}_i\beta_i}{\widetilde{\tau}_\theta^i\widetilde{\tau}_u^i(\widetilde{\Delta}^i + \hat{\beta})^2}\right)\varphi_i \\ &\quad + 1 + 2\rho\alpha_i\beta_i\bar{a}_i - \frac{(\rho\bar{a}_i\beta_i)^2}{\widetilde{\tau}_\theta^i\widetilde{\tau}_u^i(\widetilde{\Delta}^i + \hat{\beta})^2} + (2\bar{\tau}_i - \mathrm{Var}(\xi_i))\gamma_i \\ &= -\frac{\rho^2}{\widetilde{\tau}_\theta^i\widetilde{\tau}_u^i(\widetilde{\Delta}^i + \hat{\beta})^2}\varphi_i^2 - 2\rho\frac{\widetilde{\tau}_\theta^i(1 - \bar{a}_i) - \widetilde{\Delta}^i\hat{\beta}\widetilde{\tau}_u^i}{\widetilde{\tau}_\theta^i\widetilde{\tau}_u^i(\widetilde{\Delta}^i + \hat{\beta})^2}\varphi_i \\ &\quad + 1 + \widetilde{\tau}_\theta^i\frac{2\bar{a}_i(\widetilde{\tau}_\theta^i - \widetilde{\Delta}^i\hat{\beta}\widetilde{\tau}_u^i) - \bar{a}_i^2\widetilde{\tau}_\theta^i}{\widetilde{\tau}_\theta^i\widetilde{\tau}_u^i(\widetilde{\Delta}^i + \hat{\beta})^2} + \sum_{j=1}^n (2a_{ij} - a_{ij}^2)\tau_j\gamma_i. \end{aligned} \tag{37}$$

From (37), we can show by contradiction that lack of financial literacy and regret aversion under ambiguity imply $\varphi_i = 0$. Indeed, when $\varphi_i \neq 0$, the determinant in (37) becomes strictly negative and then the aggregated strategy is dominated by single suggested strategy when both $\widetilde{\tau}_\theta^i$ and $\widetilde{\tau}_u^i$ are sufficiently small. Furthermore, from (37) with the setting $\varphi_i = 0$, we see that the bounded rationality of regret aversion under ambiguity holds if and only if

$$1 + \widetilde{\tau}_\theta^i \frac{2\bar{a}_i(\widetilde{\tau}_\theta^i - \widetilde{\Delta}^i \hat{\beta} \widetilde{\tau}_u^i) - \bar{a}_i^2 \widetilde{\tau}_\theta^i}{\widetilde{\tau}_\theta^i \widetilde{\tau}_u^i (\widetilde{\Delta}^i + \hat{\beta})^2} + (2\bar{\tau}_i - \mathrm{Var}(\xi_i))\gamma_i > 1 + \widetilde{\tau}_\theta^i \frac{2(\widetilde{\tau}_\theta^i - \widetilde{\Delta}^i \hat{\beta} \widetilde{\tau}_u^i) - \widetilde{\tau}_\theta^i}{\widetilde{\tau}_\theta^i \widetilde{\tau}_u^i (\widetilde{\Delta}^i + \hat{\beta})^2} + \max_{1 \leq j \leq n} \tau_j \gamma_i,$$

which is equivalent to

$$\left(-(1 - \bar{a}_i)^2 \widetilde{\tau}_\theta^i + 2(1 - \bar{a}_i)\widetilde{\Delta}^i \hat{\beta} \widetilde{\tau}_u^i\right) \frac{1}{\widetilde{\tau}_u^i (\widetilde{\Delta}^i + \hat{\beta})^2} + \left(\sum_{j=1}^n (2a_{ij} - a_{ij}^2)\tau_j - \max_{1 \leq j \leq n} \tau_j\right) \gamma_i > 0.$$

However, the above inequality fails to hold when $\widetilde{\tau}_u^i$ is sufficiently small unless that $\bar{a}_i = 1$. Moreover, the result that conformism implies directional adherence, i.e., $a_{ij} \geq 0$ also apply here. Following the similar procedure in the proof of Proposition 1, we can also show that the matrix $\Sigma^{-1} + 2B$ is positive definite under the conditions $a_{ij} \geq 0$ and $\bar{a}_i = \sum_{j=1}^n a_{ij} = 1$. Thus, Proposition 1 in the paper also holds for the difference of opinion model.

Finally, from (37) with the setting $\varphi_i = 0$ and $\bar{a}_i = 1$, we see that the optimal aggregation problem translates to the optimization problem (11), and consequently, the optimal aggregation results in Proposition 3 also hold and there exists a unique linear equilibrium in this alternative setting. □

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