Investment and asset pricing with relative wealth concerns and multiple risky assets

Luca De Gennaro Aquino* Enrico G. De Giorgi† Youcheng Lou‡
Moris S. Strub§

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Abstract

We study optimal investment and asset pricing in a two-agent CARA utility model with relative wealth concerns and both normal and non-normal assets. We show that relative wealth concerns are equivalent to modified risk tolerance parameters across a broad class of payoff distributions. In equilibrium, agents hold identical risky asset weights, regardless of higher moments or risk aversion levels. Comparative statics reveal how portfolios, prices, and expected wealth gaps depend on the strength of relative wealth concerns, with allocations and wealth rankings fully determined by preference parameters.

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*Department of Engineering, Reykjavik University, Menntavegur 1, Reykjavik 102, Iceland; lucaa@ru.is

†Department of Economics, School of Economics and Political Science, University of St. Gallen, Rosenbergstrasse 22, St. Gallen 9000, Switzerland; enrico.degiorgi@unisg.ch

‡Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Zhongguancun East Road 55, Haidian District, Beijing 100190, China; louyoucheng@amss.ac.cn

§Warwick Business School, University of Warwick, Scarman Road, Coventry CV4 7AL, United Kingdom; moris.strub@wbs.ac.uk

1 Introduction

The concept of relative wealth concerns, where an agent's utility derived from their own consumption is influenced by the consumption levels of others, has been a significant topic of interest in investment and asset pricing since its formalization by Abel (1990) and Galí (1994). These pioneering studies showed that negative consumption externalities increase marginal rates of substitution, thereby amplifying the equity risk premium.

Subsequent research expanded on these ideas. Closely related to our paper, Gollier (2004) considered environments in which agents adjust their marginal utility by conforming (or anti-conforming) to the strategies of others; Gómez (2007) examined how portfolio choice and asset prices are affected when agents seek to outperform an exogenous benchmark; Levy and Levy (2015) studied the mean-variance efficiency and degree of diversification in portfolios selected using the peer-group reference as the market portfolio.[1]

We contribute to this literature by studying a two-agent economy in which agents exhibit relative wealth concerns and face both normally and non-normally distributed asset payoffs. Preferences are described by CARA utility functions applied to a weighted difference between their own wealth and that of the other agent.[2] Since each agent optimizes their strategy relative to the competitor's wealth, we analyze equilibria in two dimensions: the strategic equilibrium induced by reference-dependent preferences, and the market equilibrium balancing supply and demand. Our focus lies in characterizing the equilibrium portfolio strategies and the resulting asset prices.

Our main contributions are the following. First, we show that, even in the presence of non-normal assets, the market equilibrium for CARA utility maximizers with relative wealth concerns coincides with that of a hypothetical economy populated by CARA utility maximizers without relative wealth concerns but with modified risk tolerance parameters. While similar equivalence results have been noted, for instance, in the consumption-based asset pricing model

[1] See also Admati and Pfleiderer (1997), Chan and Kogan (2002), DeMarzo et al. (2008), Gebhardt (2011), Curatola (2017), Qiu (2017). For recent empirical evidence on the role of others for wealth accumulation we refer the reader to Haliassos (2024).

[2] See, among others, Breon-Drish (2015), Glebkin et al. (2020), and Chabakauri et al. (2022) for related analyses of CARA preferences with non-normal payoffs.

with externalities of Galí (1994) and Gómez (2007), or the continuous-time models with normally distributed stock returns of Espinosa and Touzi (2015) and Lacker and Zariphopoulou (2019), we show that this equivalence holds more broadly for any asset payoff distribution belonging to the natural exponential family.[3]

Second, we establish that a version of the two-fund separation theorem holds in the context of CARA utility maximization with relative wealth concerns. In equilibrium, all agents hold the same proportion of each risky asset, with portfolio weights proportional to the relative supply of the assets. This result highlights a strong form of diversification: despite heterogeneity in risk preferences and concerns over relative wealth, agents share risk in fixed proportions, leading to homogeneous portfolio compositions.

Finally, we examine the comparative statics of equilibrium strategies, asset prices, and expected wealth differences with respect to the coefficients of relative wealth concerns. In particular, we find that when the product of these coefficients, $\alpha_A \alpha_B$, is less than one, both agents are effectively risk-averse and the risky asset earns a positive premium. In contrast, when $\alpha_A \alpha_B$ exceeds one (implying that at least one agent places greater weight on the other's wealth than on their own), agents behave as risk-seekers and the risky asset offers a negative premium. As a consequence, agents' expected wealth is increasing with their degree of relative wealth concerns when $\alpha_A \alpha_B < 1$, but decreasing when $\alpha_A \alpha_B > 1$, everything else being constant.

The rest of the paper is structured as follows. In Section 2, we introduce CARA preferences with relative wealth concerns and study a baseline model with a single normally distributed security. Extended models with a binomially distributed asset and more general payoff structures are discussed in Section 3 and Section 4, respectively. Section 5 concludes. Proofs are provided in the Appendix.

[3]The natural exponential family includes the normal distribution, the Poisson distribution, the gamma distribution, and the binomial distribution. See Section 4 for details.

2 Baseline model: Normally distributed asset

The economy consists of two agents, labelled as A and B, with preferences described by a CARA utility function with relative wealth concerns:

$$\begin{aligned} u_A(W_A, W_B) &= -\exp\left(-\frac{1}{\gamma_A}(W_A - \alpha_A W_B)\right), \\ u_B(W_B, W_A) &= -\exp\left(-\frac{1}{\gamma_B}(W_B - \alpha_B W_A)\right), \end{aligned} \tag{1}$$

where W_A, W_B indicate the final wealths, γ_A, γ_B are coefficients of risk tolerance (inverse of risk aversion), and $\alpha_A, \alpha_B > 0$ are coefficients of relative wealth concerns.[4]

A key feature of the specification in (1) is that the preferences and, therefore, the associated optimal strategy, of agent A depend on the strategy of agent B through the resulting terminal wealth of agent B, and vice versa. This succinctly reflects the idea that agents evaluate their wealth (or consumption) relatively to their peers or a certain benchmark. For exponential utilities, such comparison is done in terms of dollar amounts, whereas with power utilities comparisons are typically made in terms of returns.

Remark 1. *In this context, A and B can also be interpreted as two competing groups of agents, with the agents within each group being symmetric. More precisely, agents in group A consider the weighted average wealth $\tilde{\alpha}_A W_A + (1-\tilde{\alpha}_A)W_B$ as the benchmark with a coefficient of relative wealth concerns β_A. That is, the utility of each agent in group A is given by*

$$\begin{aligned} &-\exp\left(-\frac{1}{\gamma_A}(W_A - \beta_A(\tilde{\alpha}_A W_A + (1-\tilde{\alpha}_A)W_B))\right) \\ &\qquad = -\exp\left(-\frac{1}{\gamma_A/(1-\beta_A\tilde{\alpha}_A)}\left(W_A - \frac{\beta_A(1-\tilde{\alpha}_A)}{1-\beta_A\tilde{\alpha}_A}W_B\right)\right), \end{aligned}$$

which is a special case of the model setting (1) with the interpretation that $\frac{\gamma_A}{1-\beta_A\tilde{\alpha}_A}$ is the risk tolerance coefficient and $\frac{\beta_A(1-\tilde{\alpha}_A)}{1-\beta_A\tilde{\alpha}_A}$ is the coefficient of relative wealth concerns of agents in group A. A similar justification also applies to group B.

[4] Several papers have looked at analogous preferences. In particular, Garcia and Strobl (2011) and Guo and Lou (2023) considered the case in which each agent compares her/his wealth with the average wealth in an economy populated by a continuum of agents, while Frei and Dos Reis (2011), Espinosa and Touzi (2015), Lacker and Zariphopoulou (2019), and Liang et al. (2023) studied relative performance with respect to the (arithmetic) average performance of n other agents, as well as the mean-field limit for when the number of agents n goes to infinity.

We start by considering a baseline model in which the investment opportunities are represented by one risky asset (stock) with normally distributed payoff and one risk-free asset. The risky asset is in fixed supply of z_1 units with price p_1, which yields a payoff $\tilde{v} \sim \mathcal{N}(\mu, \sigma^2)$, while the risk-free asset has a perfectly elastic supply with price and interest normalized to 1. Agents are assumed to have symmetric information and to agree on the prior distribution of the stock. Payoffs are realized at time $t = 1$.

We denote by $\theta_{i,1}$ the units of the risky asset bought by agent $i = A, B$ at time $t = 0$. Assuming zero initial wealth for both agents,[5] the final wealth is $W_i = \theta_{i,1}(\tilde{v} - p_1)$. We sometimes write $W_i^{\theta_{i,1}}$, $i = A, B$, to emphasize the dependence of the final wealth on the chosen strategy.

In this setting, equilibria are characterized by a pair of trading strategies $(\theta^*_{A,1}, \theta^*_{B,1})$ and a price p^*_1 that satisfy the following conditions:

(i) Each investor selects a trading strategy that maximizes their CARA expected utility with relative wealth concerns, given a fixed strategy of the other investor. Formally,

$$\theta^*_{A,1} = \arg\max_{\theta_{A,1}} \mathbb{E}\left[-\exp\left(-\frac{1}{\gamma_A}\left(W_A^{\theta_{A,1}} - \alpha_A W_B^{\theta^*_{B,1}}\right)\right)\right],$$

$$\theta^*_{B,1} = \arg\max_{\theta_{B,1}} \mathbb{E}\left[-\exp\left(-\frac{1}{\gamma_B}\left(W_B^{\theta_{B,1}} - \alpha_B W_A^{\theta^*_{A,1}}\right)\right)\right].$$

(ii) The market clears, that is,

$$\theta^*_{A,1} + \theta^*_{B,1} = z_1.$$

Conditions (i) and (ii) imply that an equilibrium must have two dimensions: a Nash equilibrium in agents' strategies, induced by reference dependent preferences, and a market equilibrium between supply and demand. The following proposition characterizes such an equilibrium and shows that it exists.

[5] This is a common assumption for CARA utility maximization, as optimal demands for risky asset allocation do not depend on initial wealth. In the presence of relative wealth concerns, this assumption can be interpreted as a normalization of wealth relative to the peer, ensuring that the analysis is not distorted by absolute wealth levels.

Proposition 1. *A unique, linear equilibrium exists, where the equilibrium price of the risky asset is*

$$p_1^* = \mu - \frac{z_1\,(1-\alpha_A\alpha_B)\,\sigma^2}{\gamma_A(1+\alpha_B)+\gamma_B(1+\alpha_A)}, \tag{2}$$

and the equilibrium allocations are given by

$$\begin{aligned} \theta_{A,1}^* &= \frac{\mu-p_1}{\sigma^2}\left(\frac{\gamma_A+\alpha_A\gamma_B}{1-\alpha_A\alpha_B}\right) = \frac{z_1\,(\gamma_A+\alpha_A\gamma_B)}{\gamma_A(1+\alpha_B)+\gamma_B(1+\alpha_A)},\\ \theta_{B,1}^* &= \frac{\mu-p_1}{\sigma^2}\left(\frac{\gamma_B+\alpha_B\gamma_A}{1-\alpha_A\alpha_B}\right) = \frac{z_1\,(\gamma_B+\alpha_B\gamma_A)}{\gamma_A(1+\alpha_B)+\gamma_B(1+\alpha_A)}. \end{aligned} \tag{3}$$

Let us comment on this result. First, we note that the equilibrium allocations in (3) are equivalent to the optimal investments of two agents *without* relative wealth concerns and with implied risk tolerance coefficients

$$\tilde{\gamma}_A := \frac{\gamma_A+\alpha_A\gamma_B}{1-\alpha_A\alpha_B} \quad \text{and} \quad \tilde{\gamma}_B := \frac{\gamma_B+\alpha_B\gamma_A}{1-\alpha_A\alpha_B}. \tag{4}$$

In Section 3, we will show that the equivalence between an economy with relative wealth concerns and a hypothetical economy without relative wealth concerns continues to hold with the same implied risk tolerance coefficients for assets with skewed payoffs.

Assuming for now that α_A and α_B are positive, we observe the following. When $\alpha_A\alpha_B < 1$, both agents are effectively risk-averse and the risky asset carries a positive premium; see Equation (2). Conversely, when $\alpha_A\alpha_B > 1$ –which arises when at least one agent places greater weight on the other agent's wealth than on their own– the agents exhibit risk-seeking behavior, and the risky asset offers a negative premium. Finally, as $\alpha_A\alpha_B$ tends to 1 agents become increasingly risk tolerant, thus the market moves towards a homogeneous-holdings equilibrium in which each agent holds the same fraction of the asset supply, regardless of the asset's price:

$$\theta_A^*\big|_{\alpha_A\alpha_B=1} = \theta_B^*\big|_{\alpha_A\alpha_B=1} = \frac{z_1}{2}.$$

In this case, the market-clearing price is given by the mean payoff, $p_1^* = \mu$.[6]

We, next, study the effect of the coefficient of relative wealth concerns on the optimal allocation and equilibrium price.

[6] While this is not pointed out in their paper, one can show that a similar consequence can be derived from Proposition 1 in Garcia and Strobl (2011) when the fraction of informed (versus uninformed) investors is set equal to 0; cf. the proof of Proposition 1, together with Eq. (4), therein.

Proposition 2. *The equilibrium strategy of agent A:*

(i) strictly increases (decreases) in α_A *if and only if* $\gamma_B + \alpha_B\gamma_A > (<)\, 0$*;*

(ii) strictly increases (decreases) in α_B *if and only if* $\gamma_A + \alpha_A\gamma_B > (<)\, 0$*.*

Further, the equilibrium price

(iii) strictly increases (decreases) in α_A *if and only if* $\alpha_B\gamma_A + \alpha_B\gamma_B + \alpha_B^2\gamma_A + \gamma_A > (<)\, 0$*;*

(iv) strictly decreases (increases) in α_B *if and only if* $\alpha_A\gamma_A + \alpha_A\gamma_B + \alpha_A^2\gamma_B + \gamma_B > (<)\, 0$*.*

Mutatis mutandis, the comparative statics for the equilibrium strategy of agent B hold equivalently.

The ordinal relationship of expected wealth of the two agents is given in the proposition below.

Proposition 3. *The ordinal relationship of expected wealth of the two agents is determined by their preference parameters as follows:*

$$\begin{cases} \mathbb{E}\left[W_A\right] > \mathbb{E}\left[W_B\right], & \text{if } (1-\alpha_A\alpha_B)\left((\gamma_A+\alpha_A\gamma_B)-(\gamma_B+\alpha_B\gamma_A)\right) > 0, \\ \mathbb{E}\left[W_A\right] = \mathbb{E}\left[W_B\right], & \text{if } (1-\alpha_A\alpha_B)\left((\gamma_A+\alpha_A\gamma_B)-(\gamma_B+\alpha_B\gamma_A)\right) = 0, \\ \mathbb{E}\left[W_A\right] < \mathbb{E}\left[W_B\right], & \text{if } (1-\alpha_A\alpha_B)\left((\gamma_A+\alpha_A\gamma_B)-(\gamma_B+\alpha_B\gamma_A)\right) < 0. \end{cases}$$

We conclude this section with a proposition that characterizes how the equilibrium wealth gap depends on the coefficients of relative wealth concerns, under the assumption that agents have identical risk tolerance.[7]

Proposition 4. *Assume that the two agents have identical coefficients of risk tolerance, i.e.,* $\gamma_A = \gamma_B =: \gamma$*. Then:*

(i) If $\alpha_A\alpha_B < 1$*, then* $\mathbb{E}[W_A - W_B]$ *increases with* α_A*, and is positive (negative) when* $\alpha_A > (<)\, \alpha_B$*;*

[7] For a dedicated analysis on the impact of relative wealth concerns on wealth gap in the presence of asymmetric information, we refer to Guo and Lou (2023).

(ii) If $\alpha_A\alpha_B > 1$, then $\mathbb{E}[W_A - W_B]$ decreases with α_A, and is negative (positive) when $\alpha_A >$ $(<)$ α_B;

(iii) If $\alpha_A\alpha_B = 1$, or $\alpha_A = \alpha_B$, then $\mathbb{E}[W_A - W_B] = 0$.

In addition, for a fixed α_B:

(iv) $\mathbb{E}[W_A - W_B]$ strictly increases (decreases) for $\alpha_A < (>)\ \alpha_A^* := \dfrac{\alpha_B^2 + \alpha_B + 2}{3\alpha_B + 1}$;

(v) $\mathbb{E}[W_A - W_B]$ reaches its maximum at α_A^*, where it is equal to

$$\mathbb{E}[W_A - W_B]\,\Big|_{\alpha_A=\alpha_A^*} = \frac{z_1^2\sigma^2}{\gamma} \times \frac{(\alpha_B - 1)^2}{8(\alpha_B + 1)}.$$

In line with the discussion after Proposition 1, we consider three distinct scenarios. When $\alpha_A\alpha_B < 1$, as previously noted, the risky asset offers a positive premium. In this case, agent A benefits from increasing her/his concern for relative wealth, as this induces greater risk-taking (per Eq. (4)) and raises her/his expected wealth. On the contrary, when $\alpha_A\alpha_B > 1$, the risky asset offers a negative premium, and agent A's expected wealth decreases as she/he becomes more concerned about relative wealth. Finally, when $\alpha_A\alpha_B = 1$, or when the two agents are perfectly symmetric ($\alpha_A = \alpha_B$), the expected wealth gap vanishes, as the agents match each other's behavior exactly.

3 Extended model: Skewed asset

In this section, we present an extension of the baseline model in which the investment opportunities include a positively skewed asset. Following Barberis and Huang (2008), we assume a binomially distributed payoff $\widetilde{L} \sim (J, q; 0, 1-q)$ that is uncorrelated to the existing asset $\tilde{v}$. In other words, akin to a lottery ticket or a binary call option, $\widetilde{L}$ pays J with probability $q \in (0, 1)$ and 0 otherwise.

We denote by $\theta_{i,1}$ and $\theta_{i,2}$ the allocation of agent $i = A, B$ in the securities with payoff $\tilde{v}$ and $\widetilde{L}$, respectively. Prices are denoted by p_1 and p_2, and the fixed supplies are denoted by z_1 and z_2.

Equilibria are now characterized by a 4-tuple of trading strategies $(\theta_{A,1}^*, \theta_{A,2}^*, \theta_{B,1}^*, \theta_{B,2}^*)$ and prices (p_1^*, p_2^*) satisfying the following conditions:

(i) Each investor selects a trading strategy that maximizes her/his CARA expected utility with relative wealth concerns, given a fixed strategy of the other investor. The optimization problems are revised as follows:

$$\max_{\theta_{A,1},\theta_{A,2}} \mathbb{E}\left[-\exp\left(-\frac{1}{\gamma_A}\left(W_A-\alpha_A W_B\right)\right)\right],$$

$$\max_{\theta_{B,1},\theta_{B,2}} \mathbb{E}\left[-\exp\left(-\frac{1}{\gamma_B}\left(W_B-\alpha_B W_A\right)\right)\right],$$

where $W_i=\theta_{i,1}(\tilde{v}-p_1)+\theta_{i,2}\big(\widetilde{L}-p_2\big)$, $i=A,B$.

(ii) The market clears for each asset, i.e.,

$$\theta^*_{A,1}+\theta^*_{B,1}=z_1,$$

$$\theta^*_{A,2}+\theta^*_{B,2}=z_2.$$

The following proposition characterizes the equilibrium and shows that it exists.

Proposition 5. *A unique equilibrium exists, where the equilibrium prices of the risky assets are*

$$\begin{aligned} p_1^* &= \mu-\frac{z_1\left(1-\alpha_A\alpha_B\right)\sigma^2}{\gamma_A(1+\alpha_B)+\gamma_B(1+\alpha_A)}, \\ p_2^* &= \frac{qJ}{q+(1-q)\exp\left(\dfrac{z_2 J(1-\alpha_A\alpha_B)}{\gamma_A(1+\alpha_B)+\gamma_B(1+\alpha_A)}\right)}, \end{aligned} \tag{5}$$

and the equilibrium allocations in the risky assets are given by

$$\begin{aligned} \theta^*_{A,1} &= \frac{\mu-p_1}{\sigma^2}\left(\frac{\gamma_A+\alpha_A\gamma_B}{1-\alpha_A\alpha_B}\right)=\frac{z_1\left(\gamma_A+\alpha_A\gamma_B\right)}{\gamma_A(1+\alpha_B)+\gamma_B(1+\alpha_A)}, \\ \theta^*_{A,2} &= \frac{1}{J}\left(\frac{\gamma_A+\alpha_A\gamma_B}{1-\alpha_A\alpha_B}\right)\log\left(\frac{q(J-p_2)}{(1-q)p_2}\right)=\frac{z_2(\gamma_A+\alpha_A\gamma_B)}{\gamma_A(1+\alpha_B)+\gamma_B(1+\alpha_A)}, \\ \theta^*_{B,1} &= \frac{\mu-p_1}{\sigma^2}\left(\frac{\gamma_B+\alpha_B\gamma_A}{1-\alpha_A\alpha_B}\right)=\frac{z_1\left(\gamma_B+\alpha_B\gamma_A\right)}{\gamma_A(1+\alpha_B)+\gamma_B(1+\alpha_A)}, \\ \theta^*_{B,2} &= \frac{1}{J}\left(\frac{\gamma_B+\alpha_B\gamma_A}{1-\alpha_A\alpha_B}\right)\log\left(\frac{q(J-p_2)}{(1-q)p_2}\right)=\frac{z_2(\gamma_B+\alpha_B\gamma_A)}{\gamma_A(1+\alpha_B)+\gamma_B(1+\alpha_A)}. \end{aligned} \tag{6}$$

An interesting outcome of the model is the following corollary, which shows that, in the absence of frictions, agents' portfolio diversification levels are identical.[8]

Corollary 1. *Observing in (6) the strategies evaluated at the market-clearing price, it turns out that, in equilibrium, agents with possibly different levels of risk attitudes and relative wealth concerns own portfolios with similar composition. Namely, the ratio between the number of shares held in each asset by A and B is equal to the ratio of the respective supplies:*

$$\frac{\theta^*_{A,1}}{\theta^*_{A,2}} = \frac{\theta^*_{B,1}}{\theta^*_{B,2}} = \frac{z_1}{z_2}.$$

The classical two-fund separation theorem states that all mean-variance investors hold a combination of the risk-free asset and a single fund of risky assets, the tangency portfolio. Portfolio choice thus separates into identifying the optimal portfolio of risky assets and determining the allocation between the risky fund and the risk-free asset, which might differ across investors due to heterogeneity in their risk tolerance. Corollary 1 shows that the number of shares held in each asset is the same for both investors. This can thus be interpreted as an extension of the two-fund separation theorem to a CARA setting with relative wealth concerns.

From (6), we observe that relative wealth concerns lead to exactly the same change in implied risk tolerance (see (4)) for both normally and binomially distributed assets. That is, restricting to the more standard case with $\alpha_A\alpha_B < 1$, agents with exponential utility who are concerned by peers increase their risk tolerance consistently across assets, regardless of the skewness. One could then wonder whether this is true for more general distributions. In Section 4, we briefly argue that this is the case for at least all distributions in the *natural exponential family* (NEF).[9]

[8] In a model with "keeping up with the Joneses" utility of consumption, Gómez (2007) shows that no non-diversification equilibria are possible when the utility function $u(c_i)$ of each agent i satisfies the condition $-\frac{u'(c_i)}{u''(c_i)} = A_i + Bc_i$, for constants A_i and B; see Proposition 1 therein. As CARA utilities satisfy the above condition, our result characterizes the (non-diversification) equilibrium in the case of multiple assets with different distributions.

[9] See Morris (1982) and Chapter 3 in Casella and Berger (2002) for a description of the NEF, and also Breon-Drish (2015), who studies a noisy rational expectations model in which the risky asset follows (conditionally on the signal received by the agent) a distribution from a related exponential family.

Next, we study the effect of the coefficient of relative wealth concerns on the optimal allocations and equilibrium price of the skewed security. The results for the normally distributed security carry forward from Proposition 2.

Proposition 6. *The equilibrium strategy of agent A in the skewed asset:*

(i) strictly increases (decreases) in α_A *if and only if* $\gamma_B + \alpha_B \gamma_A > (<)\, 0$*;*

(ii) strictly increases (decreases) in α_B *if and only if* $\gamma_A + \alpha_A \gamma_B > (<)\, 0$*.*

Further, the equilibrium price of the skewed security:

(iii) strictly increases (decreases) in α_A *if and only if* $\alpha_B\gamma_A + \alpha_B\gamma_B + \alpha_B^2\gamma_A + \gamma_A > (<)\, 0$*;*

(iv) strictly decreases (increases) in α_B *if and only if* $\alpha_A\gamma_A + \alpha_A\gamma_B + \alpha_A^2\gamma_B + \gamma_B > (<)\, 0$*.*

The comparative statics for the equilibrium strategy of agent B hold equivalently, with the due changes in notation.

Remark 2. *Echoing our earlier comment about the changes in risk tolerance, we note that the conditions in Proposition 6 are equal to those in Proposition 2.*

Interestingly, the ordinal relationship of expected wealth of the two agents is again fully determined by their preference parameters and the same as in the baseline model (Proposition 3).

Proposition 7. *The ordinal relationship of expected wealth of the two agents is determined by their preference parameters as follows:*

$$\begin{cases} \mathbb{E}\left[W_A\right] > \mathbb{E}\left[W_B\right], & \text{if } (1-\alpha_A\alpha_B)\left((\gamma_A+\alpha_A\gamma_B)-(\gamma_B+\alpha_B\gamma_A)\right) > 0, \\ \mathbb{E}\left[W_A\right] = \mathbb{E}\left[W_B\right], & \text{if } (1-\alpha_A\alpha_B)\left((\gamma_A+\alpha_A\gamma_B)-(\gamma_B+\alpha_B\gamma_A)\right) = 0, \\ \mathbb{E}\left[W_A\right] < \mathbb{E}\left[W_B\right], & \text{if } (1-\alpha_A\alpha_B)\left((\gamma_A+\alpha_A\gamma_B)-(\gamma_B+\alpha_B\gamma_A)\right) < 0. \end{cases}$$

Similar to Proposition 4, the next result characterizes the dependence of the wealth gap on relative wealth concerns when agents have the same risk tolerance. The conclusion mirrors the baseline case.

Proposition 8. *Assume that the two agents have identical coefficients of risk tolerance, i.e., $\gamma_A = \gamma_B = \gamma$. Then:*

(i) If $\alpha_A \alpha_B < 1$, then $\mathbb{E}[W_A - W_B]$ increases with α_A, and is positive (negative) when $\alpha_A >$ $(<)$ α_B;

(ii) If $\alpha_A \alpha_B > 1$, then $\mathbb{E}[W_A - W_B]$ decreases with α_A, and is negative (positive) when $\alpha_A >$ $(<)$ α_B;

(iii) If $\alpha_A \alpha_B = 1$, or $\alpha_A = \alpha_B$, then $\mathbb{E}[W_A - W_B] = 0$.

4 Payoffs with distribution from a natural exponential family

In this final section, we characterize the market equilibrium when assets belong to the *natural exponential family (NEF)*. While a complete analysis is beyond the scope herein, we show that some of the main features identified in previous sections extend to this more general setting.

First, we recall that, for a given parameter η, the natural exponential family includes distributions whose probability density function can be expressed as

$$f(x \mid \eta) = h(x) \exp\big(\eta x - g(\eta)\big),$$

with known functions $h(x)$ and $g(\eta)$.

Now consider two uncorrelated risky assets $\widetilde{X}^1$ and $\widetilde{X}^2$, with price $p_{\widetilde{X}^1}$ and $p_{\widetilde{X}^2}$, respectively, following a distribution within the NEF. By using the moment generating function of $\widetilde{X}^1$ and $\widetilde{X}^2$,

$$\mathbb{E}\left[\exp\left(u\widetilde{X}^j\right)\right] = \exp\Big(g_j(\eta_j + u) - g_j(\eta_j)\Big), \quad j = 1, 2, \ u \in \mathbb{R},$$

it is easy to show that the demand functions of agent A and B for these assets are given by

$$\theta_{A,j} = \gamma_A\Big(\eta_j - (g'_j)^{-1}(p_{\widetilde{X}^j})\Big) + \alpha_A \theta_{B,j},$$

$$\theta_{B,j} = \gamma_B\Big(\eta_j - (g'_j)^{-1}(p_{\widetilde{X}^j})\Big) + \alpha_B \theta_{A,j},$$

where the assumption is that $g'_j, j = 1, 2$, is invertible and the inverse well-defined at $p_{\widetilde{X}^j}$.

Therefore, the equilibrium allocations are

$$\theta^*_{A,j} = \frac{\gamma_A + \alpha_A\gamma_B}{1 - \alpha_A\alpha_B}\Big(\eta_j - (g'_j)^{-1}(p_{\widetilde{X}^j})\Big),$$
$$\theta^*_{B,j} = \frac{\gamma_B + \alpha_B\gamma_A}{1 - \alpha_A\alpha_B}\Big(\eta_j - (g'_j)^{-1}(p_{\widetilde{X}^j})\Big),$$

which shows that the implied coefficients of risk tolerance are equivalent for both assets.

Furthermore, supposing that the two assets are supplied in z_1 and z_2 units, the market-clearing prices are derived as follows:

$$p^*_{\widetilde{X}^j} = (g'_j)\left(\eta_j - \frac{z_j(1 - \alpha_A\alpha_B)}{\gamma_A + \gamma_B + \alpha_A\gamma_B + \alpha_B\gamma_A}\right).$$

5 Concluding remarks

Neighbors, peers, and other exogenous benchmarks have proved to exert a substantial influence on individual wealth accumulation, both through their characteristics and their behavior (Haliassos (2024)). Within this context, we examine the role of relative wealth concerns in a two-agent economy with CARA preferences and multiple risky assets. Our analysis shows that, in equilibrium, optimal investment choices and the ranking of expected wealth outcomes are entirely driven by the preference parameters. In particular, if both agents are only partly concerned about the other agent's wealth, risky assets offer a positive premium and thus greater risk-taking leads to higher expected wealth; conversely, if at least one agent is significantly more concerned with the other agent's wealth than her/his own, risky assets turn out to offer a negative premium and thus more risk-taking leads to a lower expected wealth. Moreover, we found that a two-fund separation holds, whereby agents select identical portfolio compositions -characterized by constant allocation ratios across assets- regardless of the statistical properties of asset returns and heterogeneity in agents' risk tolerance. To further refine these insights, future research could explore richer economic environments, for instance by adopting behavioral preferences or agent heterogeneity.

Appendix A. Proofs

A.1 Proof of Proposition 1

The problem being symmetric, we show the solution only from the point of view of agent A. First, we write the relative wealth of agent A:

$$\begin{aligned} W_A - \alpha_A W_B &= \theta_A \tilde{v} - \theta_A p_1 - \alpha_A \left(\theta_B \tilde{v} - \theta_B p_1\right) \\ &= \left(\theta_A - \alpha_A \theta_B\right) \left(\tilde{v} - p_1\right). \end{aligned}$$

By the assumption of normality of the payoff v, it follows that

$$W_A - \alpha_A W_B \sim \mathcal{N}\left(\left(\theta_A - \alpha_A \theta_B\right)\left(\mu - p_1\right), \left(\theta_A - \alpha_A \theta_B\right)^2 \sigma^2\right).$$

The maximization problem of agent A, for a fixed strategy of agent B, reads:

$$\begin{aligned} &\max_{\theta_A} \mathbb{E}\left[-\exp\left(-\frac{1}{\gamma_A}\left(W_A - \alpha_A W_B\right)\right)\right] \\ &= \max_{\theta_A} \ -\exp\left(-\frac{1}{\gamma_A}\left(\mathbb{E}\left[W_A - \alpha_A W_B\right] - \frac{1}{2\gamma_A}\mathrm{Var}\left[W_A - \alpha_A W_B\right]\right)\right) \\ &= \max_{\theta_A} -\exp\left(-\frac{1}{\gamma_A}\left(\left(\theta_A - \alpha_A\theta_B\right)\left(\mu - p_1\right) - \frac{1}{2\gamma_A}\left(\theta_A - \alpha_A \theta_B\right)^2 \sigma^2\right)\right). \end{aligned}$$

Applying the first-order condition for optimality, we obtain the optimal strategy of agent A, given the strategy of agent B:

$$\theta_A = \frac{\mu - p_1}{\frac{1}{\gamma_A}\sigma^2} + \alpha_A \theta_B. \tag{A.1}$$

Specularly, the optimal strategy of agent B, given the strategy of agent A, is

$$\theta_B = \frac{\mu - p_1}{\frac{1}{\gamma_B}\sigma^2} + \alpha_B \theta_A. \tag{A.2}$$

Replacing (A.1) into (A.2) leads to the unique equilibrium allocation:

$$\begin{aligned} \theta_A^* &= \frac{\mu - p_1}{\frac{1}{\gamma_A}\sigma^2}\left(\frac{1 + \alpha_A \frac{\gamma_B}{\gamma_A}}{1 - \alpha_A \alpha_B}\right), \\ \theta_B^* &= \frac{\mu - p_1}{\frac{1}{\gamma_B}\sigma^2}\left(\frac{1 + \alpha_B \frac{\gamma_A}{\gamma_B}}{1 - \alpha_B \alpha_A}\right). \end{aligned}$$

Finally, from the market-clearing condition, and recalling that the risky asset is supplied in z_1 units, we obtain the equilibrium price:

$$\begin{aligned}\theta_A^* + \theta_B^* = z_1 &\iff \frac{\mu - p_1}{\frac{1}{\gamma_A}\sigma^2}\left(\frac{1 + \alpha_A \frac{\gamma_B}{\gamma_A}}{1 - \alpha_A \alpha_B}\right) + \frac{\mu - p_1}{\frac{1}{\gamma_B}\sigma^2}\left(\frac{1 + \alpha_B \frac{\gamma_A}{\gamma_B}}{1 - \alpha_B \alpha_A}\right) = z_1 \\ &\iff p_1 = \mu - \frac{z_1 \left(1 - \alpha_A \alpha_B\right) \sigma^2}{\gamma_A (1 + \alpha_B) + \gamma_B (1 + \alpha_A)}.\end{aligned}$$

□

A.2 Proof of Proposition 2

To study the effect of the coefficient of relative wealth concerns on the quantities of interest, we compute the following first-order derivatives:

$$\begin{aligned}\frac{\partial \theta_A^*}{\partial \alpha_A} &= \frac{z_1 \gamma_B \left(\gamma_B + \alpha_B \gamma_A\right)}{\left(\gamma_A + \gamma_B + \alpha_A \gamma_B + \alpha_B \gamma_A\right)^2} \\ \frac{\partial \theta_A^*}{\partial \alpha_B} &= -\frac{z_1 \gamma_A \left(\gamma_A + \alpha_A \gamma_B\right)}{\left(\gamma_A + \gamma_B + \alpha_A \gamma_B + \alpha_B \gamma_A\right)^2}, \\ \frac{\partial p_1}{\partial \alpha_A} &= \sigma^2 \frac{\alpha_B \left(\gamma_A + \gamma_B + \gamma_A \alpha_B\right) + \gamma_B}{\left(\gamma_A + \gamma_B + \alpha_A \gamma_B + \alpha_B \gamma_A\right)^2}, \\ \frac{\partial p_1}{\partial \alpha_B} &= \sigma^2 \frac{\alpha_A \left(\gamma_A + \gamma_B + \gamma_B \alpha_A\right) + \gamma_A}{\left(\gamma_A + \gamma_B + \alpha_A \gamma_B + \alpha_B \gamma_A\right)^2}.\end{aligned}$$

The claim of the proposition follows immediately. □

A.3 Proof of Proposition 3

From the equilibrium price and allocations in (2)-(3), we compute

$$\begin{aligned}\mathbb{E}\left[W_A - W_B\right] &= \frac{(\mu - p_1^*)^2}{\sigma^2} \times \frac{\gamma_A + \alpha_A \gamma_B - \gamma_B - \alpha_B \gamma_A}{1 - \alpha_A \alpha_B} \\ &= z_1^2 \sigma^2 \times \frac{\left(1 - \alpha_A \alpha_B\right) \left(\gamma_A + \alpha_A \gamma_B - \gamma_B - \alpha_B \gamma_A\right)}{\left(\gamma_A + \gamma_B + \gamma_A \alpha_B + \gamma_B \alpha_A\right)^2}.\end{aligned} \tag{A.3}$$

The claim easily follows. □

A.4 Proof of Proposition 4

Setting $\gamma_A = \gamma_B =: \gamma$ in (A.3), we have

$$\mathbb{E}\left[W_A - W_B\right] = \frac{z_1^2\sigma^2}{\gamma} \times \frac{\left(1-\alpha_A\alpha_B\right)\left(\alpha_A - \alpha_B\right)}{\left(\alpha_A+\alpha_B+2\right)^2}.$$

Claims (i)-(iii) follow immediately.

Next, keeping α_B fixed, we assess

$$\frac{\partial \mathbb{E}\left[W_A - W_B\right]}{\partial \alpha_A} = \frac{z_1^2\sigma^2}{\gamma} \times \frac{(\alpha_B+1)(-\alpha_A(3\alpha_B+1)+\alpha_B^2+\alpha_B+2)}{\left(\alpha_A+\alpha_B+2\right)^3},$$

from which claims (iv)-(v) also follow easily. □

A.5 Proof of Proposition 5

As the problem is symmetric, we can consider the point of view of agent A and write her/his relative wealth as follows:

$$\begin{aligned} W_A - \alpha_A W_B &= \theta_{A,1}\tilde{v} + \theta_{A,2}\widetilde{L} - \theta_{A,1}p_1 - \theta_{A,2}p_2 \\ &\quad - \alpha_A\left(\theta_{B,1}\tilde{v} + \theta_{B,2}\widetilde{L} - \theta_{B,1}p_1 - \theta_{B,2}p_2\right) \\ &= \left(\theta_{A,1} - \alpha_A\theta_{B,1}\right)\left(\tilde{v} - p_1\right) + \left(\theta_{A,2} - \alpha_A\theta_{B,2}\right)\left(\widetilde{L} - p_2\right). \end{aligned}$$

If v and $\widetilde{L}$ are assumed to be independent, we can separate the expectation of relative wealth as follows:

$$\begin{aligned} &\max_{\theta_{A,1},\theta_{A,2}} \mathbb{E}\left[-\exp\left(-\frac{1}{\gamma_A}\left(\left(\theta_{A,1}-\alpha_A\theta_{B,1}\right)\left(\tilde{v}-p_1\right) + \left(\theta_{A,2}-\alpha_A\theta_{B,2}\right)\left(\widetilde{L}-p_2\right)\right)\right)\right] \\ &= \max_{\theta_{A,1},\theta_{A,2}} -\mathbb{E}\left[\exp\left(-\frac{1}{\gamma_A}\left(\theta_{A,1}-\alpha_A\theta_{B,1}\right)\left(\tilde{v}-p_1\right)\right)\right] \\ &\qquad \times \mathbb{E}\left[\exp\left(-\frac{1}{\gamma_A}\left(\theta_{A,2}-\alpha_A\theta_{B,2}\right)\left(\widetilde{L}-p_2\right)\right)\right]. \end{aligned} \tag{A.4}$$

We dealt with the first expectation in the proof of Proposition 1. For the second, we have:

$$\begin{aligned} &\mathbb{E}\left[\exp\left(-\frac{1}{\gamma_A}\left(\theta_{A,2}-\alpha_A\theta_{B,2}\right)\left(\widetilde{L}-p_2\right)\right)\right] \\ &= \exp\left(\frac{1}{\gamma_A}\left(\theta_{A,2}-\alpha_A\theta_{B,2}\right)p_2\right)\mathbb{E}\left[\exp\left(-\frac{1}{\gamma_A}\left(\theta_{A,2}-\alpha_A\theta_{B,2}\right)\widetilde{L}\right)\right] \\ &= \exp\left(\frac{1}{\gamma_A}\left(\theta_{A,2}-\alpha_A\theta_{B,2}\right)p_2\right)\left(q\exp\left(-\frac{1}{\gamma_A}\left(\theta_{A,2}-\alpha_A\theta_{B,2}\right)J\right)+1-q\right). \end{aligned}$$

We now conjecture (and later verify) that $J > p_2$. Applying the first order condition for optimality in (A.4), we obtain the optimal strategies of agent A, given the strategies of agent B:

$$\begin{aligned} \theta_{A,1} &= \frac{\mu - p_1}{\frac{1}{\gamma_A}\sigma^2} + \alpha_A \theta_{B,1}, \\ \theta_{A,2} &= \frac{\gamma_A}{J} \log\left(\frac{q(J-p_2)}{(1-q)p_2}\right) + \alpha_A \theta_{B,2}. \end{aligned} \tag{A.5}$$

Similarly, we can write the optimal strategies for agent B, given the strategies of agent A:

$$\begin{aligned} \theta_{B,1} &= \frac{\mu - p_1}{\frac{1}{\gamma_B}\sigma^2} + \alpha_B \theta_{A,1}, \\ \theta_{B,2} &= \frac{\gamma_B}{J} \log\left(\frac{q(J-p_2)}{(1-q)p_2}\right) + \alpha_B \theta_{A,2}. \end{aligned} \tag{A.6}$$

Replacing (A.5) into (A.6) leads to the unique equilibrium allocations:

$$\begin{aligned} \theta^*_{A,1} &= \frac{\mu - p_1}{\sigma^2(1-\alpha_A\alpha_B)}\Big(\gamma_A + \alpha_A\gamma_B\Big), \\ \theta^*_{B,1} &= \frac{\mu - p_1}{\sigma^2(1-\alpha_A\alpha_B)}\Big(\gamma_B + \alpha_B\gamma_A\Big), \\ \theta^*_{A,2} &= \frac{1}{J(1-\alpha_A\alpha_B)}(\gamma_A + \alpha_A\gamma_B)\log\left(\frac{q(J-p_2)}{(1-q)p_2}\right), \\ \theta^*_{B,2} &= \frac{1}{J(1-\alpha_A\alpha_B)}(\gamma_B + \alpha_B\gamma_A)\log\left(\frac{q(J-p_2)}{(1-q)p_2}\right). \end{aligned}$$

From the market-clearing condition, and recalling that the assets are supplied in z_1 and z_2 units, respectively, we obtain equilibrium prices:

$$\begin{aligned} z_1 = \theta^*_{A,1} + \theta^*_{B,1} &= \frac{\mu - p_1}{\frac{1}{\gamma_A}\sigma^2}\left(\frac{1 + \alpha_A\frac{\gamma_B}{\gamma_A}}{1-\alpha_A\alpha_B}\right) + \frac{\mu - p_1}{\frac{1}{\gamma_B}\sigma^2}\left(\frac{1 + \alpha_B\frac{\gamma_A}{\gamma_B}}{1-\alpha_B\alpha_A}\right) \\ &\iff p_1 = \mu - \frac{z_1\,(1-\alpha_A\alpha_B)\,\sigma^2}{\gamma_A(1+\alpha_B) + \gamma_B(1+\alpha_A)}, \\ z_2 = \theta^*_{A,2} + \theta^*_{B,2} &= \frac{1}{J(1-\alpha_A\alpha_B)}\,(\gamma_A + \gamma_B + \alpha_A\gamma_B + \alpha_B\gamma_A)\log\left(\frac{q(J-p_2)}{(1-q)p_2}\right), \\ &\iff p_2 = \frac{qJ}{q + (1-q)\exp\left(\frac{z_2 J(1-\alpha_A\alpha_B)}{\gamma_A + \gamma_B + \alpha_A\gamma_B + \alpha_B\gamma_A}\right)}. \end{aligned}$$

Finally, it remains to verify that $J > p_2$. To do so, we need to check that

$$J > \frac{qJ}{q + (1-q)\exp\left(\frac{z_2 J(1-\alpha_A\alpha_B)}{\gamma_A + \gamma_B + \alpha_A\gamma_B + \alpha_B\gamma_A}\right)}.$$

Since $J > 0$, this inequality is equivalent to

$$q + (1-q)\exp\left(\frac{z_2 J(1-\alpha_A\alpha_B)}{\gamma_A + \gamma_B + \alpha_A\gamma_B + \alpha_B\gamma_A}\right) > q,$$

which clearly holds for any $q \in (0,1)$.

□

A.6 Proof of Proposition 6

Similarly to Proposition 2, we compute the following first-order derivatives:

$$\frac{\partial\theta^*_{A,2}}{\partial\alpha_A} = \frac{z_2\left(\gamma_B^2 + \alpha_A\gamma_A\gamma_B\right)}{\left(\gamma_A + \gamma_B + \alpha_A\gamma_B + \alpha_B\gamma_A\right)^2},$$

$$\frac{\partial\theta^*_{A,2}}{\partial\alpha_B} = -\frac{z_2\left(\gamma_A^2 + \alpha_A\gamma_A\gamma_B\right)}{\left(\gamma_A + \gamma_B + \alpha_A\gamma_B + \alpha_B\gamma_A\right)^2},$$

$$\begin{aligned}\frac{\partial p_2^*}{\partial\alpha_A} = &-\frac{qJ}{\left(q + (1-q)\exp\left(\frac{z_2 J(1-\alpha_A\alpha_B)}{\gamma_A + \gamma_B + \alpha_A\gamma_B + \alpha_B\gamma_A}\right)\right)^2}\\ &\times (1-q)\exp\left(\frac{z_2 J(1-\alpha_A\alpha_B)}{\gamma_A + \gamma_B + \alpha_A\gamma_B + \alpha_B\gamma_A}\right)\\ &\times \frac{-z_2 J\left(\alpha_B\left(\gamma_A + \gamma_B + \alpha_B\gamma_A\right) + \gamma_B\right)}{\left(\gamma_A + \gamma_B + \alpha_A\gamma_B + \alpha_B\gamma_A\right)^2},\end{aligned}$$

$$\begin{aligned}\frac{\partial p_2^*}{\partial\alpha_B} = &-\frac{qJ}{\left(q + (1-q)\exp\left(\frac{z_2 J(1-\alpha_A\alpha_B)}{\gamma_A + \gamma_B + \alpha_A\gamma_B + \alpha_B\gamma_A}\right)\right)^2}\\ &\times (1-q)\exp\left(\frac{z_2 J(1-\alpha_A\alpha_B)}{\gamma_A + \gamma_B + \alpha_A\gamma_B + \alpha_B\gamma_A}\right)\\ &\times \frac{-z_2 J\left(\alpha_A\left(\gamma_A + \gamma_B + \alpha_A\gamma_B\right) + \gamma_A\right)}{\left(\gamma_A + \gamma_B + \alpha_A\gamma_B + \alpha_B\gamma_A\right)^2}.\end{aligned}$$

The claim of the proposition follows immediately. □

A.7 Proof of Proposition 7

From the equilibrium prices and allocations in (5)-(6), we compute

$$
\begin{aligned}
\mathbb{E}\left[W_A - W_B\right] &= \left(\frac{(\mu - p_1^*)^2}{\sigma^2} + \frac{1}{J}\log\left(\frac{q(J-p_2^*)}{(1-q)p_2^*}\right)(qJ - p_2^*)\right) \times \frac{\gamma_A + \alpha_A\gamma_B - \gamma_B - \alpha_B\gamma_A}{1-\alpha_A\alpha_B} \\
&= z_1^2\sigma^2 \times \frac{(1-\alpha_A\alpha_B)(\gamma_A + \alpha_A\gamma_B - \gamma_B - \alpha_B\gamma_A)}{(\gamma_A + \gamma_B + \gamma_A\alpha_B + \gamma_B\alpha_A)^2} \\
&\quad + z_2 qJ \times \frac{\gamma_A + \alpha_A\gamma_B - \gamma_B - \alpha_B\gamma_A}{\gamma_A + \gamma_B + \alpha_A\gamma_B + \alpha_B\gamma_A} \times \left(1 - \frac{1}{q + (1-q)\exp\left(\frac{z_2 J(1-\alpha_A\alpha_B)}{\gamma_A+\gamma_B}\right)}\right).
\end{aligned} \tag{A.7}
$$

Note that

$$
\operatorname{sign}(1-\alpha_A\alpha_B) = \operatorname{sign}\left(z_2 qJ\left(1 - \frac{1}{q + (1-q)\exp\left(\frac{z_2 J(1-\alpha_A\alpha_B)}{\gamma_A+\gamma_B}\right)}\right)\right).
$$

We therefore have that

$$
\operatorname{sign}\left(\mathbb{E}\left[W_A - W_B\right]\right) = \operatorname{sign}\left((1-\alpha_A\alpha_B)(\gamma_A + \alpha_A\gamma_B - \gamma_B - \alpha_B\gamma_A)\right)
$$

as claimed. □

A.8 Proof of Proposition 8

Setting $\gamma_A = \gamma_B =: \gamma$ in (A.7), we have

$$
\begin{aligned}
\mathbb{E}\left[W_A - W_B\right] &= \frac{z_1^2\sigma^2}{\gamma} \times \frac{(1-\alpha_A\alpha_B)(\alpha_A - \alpha_B)}{(2+\alpha_A+\alpha_B)^2} \\
&\quad + z_2 qJ \times \frac{\alpha_A - \alpha_B}{2+\alpha_A+\alpha_B} \times \left(1 - \frac{1}{q + (1-q)\exp\left(\frac{z_2 J(1-\alpha_A\alpha_B)}{2\gamma}\right)}\right).
\end{aligned}
$$

Claims (i)-(iii) follow immediately. □

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